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ABSTRACT

GRADES OR AGES: Grades 1-6. SUBJECT MATTER: Mathematics. ORGANIZATION AND PHYSICAL APPEARANCE: The introductory material includes an explanation of the new approach o teaching math and diagnostic tests (mathematics essentials inventories) for each grade. The main body of the guide deals with the concepts and contents of the program under the following headings: numbers and numerals, sets and sentences, whole numbers, rational numbers, measurement and money, geometry, other systems and bases, and problem solving. Each topic is subdivided by grade level. A final section deals with precision in the use of the mathematics vocabulary. The guide is mimeographed and spiral bound with a soft cover. OBJECTIVES AND ACTIVITIES: The overall objectives are listed at the beginning of the section on concepts and content, with more detailed objectives in the subdivisions of each topic. Sample activities are given on all topics for each grade. INSTRUCTIONAL MATERIALS: Bibliographies for children and teachers are provided. STUDENT ASSESSMENT: The mathematics essentials inventories are intended to be used as diagnostic and evaluative instruments. (MBM)

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MATHEMATICS IN THE ELEMENTARY SCHOOL

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Gary, Indiana
1967

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ACKNOWLEDGMENTS

I wish to express appreciation to the members of the Elementary Mathematics Materials Committee for their extra effort in the preparation of this guide. The guide is a composite of materials which have been developed previously, combined with new material. Much of the material presented in this guide is the result of their intensive work and effort.

John W. Starr 3rd.
Elementary Supervisor

PREFACE

The teaching of mathematics in the elementary school is a responsibility of major significance. Through our efforts pupils should be helped to gain an understanding of the number system in the development of our culture. Likewise, we should emphasize the development of the ability to write and recognize social uses of numbers in daily life. In developing the ability to compute accurately and quickly in mathematics, the pupils must also have a complete understanding of the processes involved.

There is a need to continuously improve teaching and learning in mathematics. New materials of instruction, new teaching approaches, and the continuing responsibility to meet the individual needs of students place great demands on all professional staff members to appraise the quality of teaching and learning in mathematics. This publication represents an effort on the part of staff members within our school system to assist all staff members in improving the teaching and learning in mathematics. It is hoped that all staff members who use this publication will find it to be of value.

Norman R Turchan
Norman R. Turchan
General Elementary Supervisor

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INTRODUCTION

The New Approach

In elementary school mathematics classes all over the country, children are working with tremendous fervor and excitement. Not only are these children responding with an eagerness which contrasts sharply with the usual classroom atmosphere, but also they are handling the familiar concepts and processes of elementary school mathematics in ways which have traditionally been the province of advanced college or graduate school courses. This seems to characterize the new mathematics.

This modern approach appears to be the result of two trends and to have been accelerated by the political and social situations of which we have rather suddenly become aware in very recent years. The first of these trends, which has been important at least since the end of World War II, has been that the teaching of arithmetic should be meaningful. In this view, which stems from psychologies holding that the most effective and efficient learning is that which emphasizes structure and understanding, elementary school mathematics is seen as a body of ideas - of concepts and relationships - to be understood. The tool subject and analytical drill approaches to the content are denied, and the social utility aspect of mathematics is less emphasized.

Arithmetic is conceived, first, as working with ideas; secondly, as getting right answers. Effective practice and mastery of skills are held to depend on an understanding of the meaning of numerals and of arithmetical processes.

The second trend comes from the discovery, by trained mathematicians, that the genesis of modes of thought of most students is in the earlier years of life and particularly during the elementary school years. Consequently, among mathematicians, there has been a growing interest in the revision of introductory college programs, high school curricula and methods, and finally in what happens in the elementary school. Also the opinion has developed that the learning of mathematics through all levels of instruction can be facilitated by a curricular plan which would involve a continuous and progressive development of mathematical ideas. The impact of this on elementary teaching has resulted in attempts to devise for the familiar arithmetic, interpretations which are more respectable mathematically and which are at the same time comprehensible to children. In addition there have been efforts to add to the curriculum elements of other traditional mathematics subjects which deal with content and ideas familiar to children in their ordinary environments. This last is the reason for interest in geometry, logic, and sets in the elementary grades.

Consequently, the efforts of the mathematics educators in emphasizing meaningful arithmetic, combine with the more intellectualized analysis of the mathematicians to produce a new view of the familiar grade school mathematics. For the elementary school, modern mathematics is no more than that, including ideas from the study of geometry, logic, and sets. It is characterized by teaching methods which attempt to establish, in the learner, the notion that the true significance of arithmetic today is as a source of pattern -- the patterns of numbers leading to higher mathematics. The methods are based largely on a discovery approach, assuming that, in the words of Jerome Bruner, "for a child to be stimulated by his environment he must be armed with the expectancy that there is something to find out."

Classroom experience with the new mathematics in the elementary school suggests that it is learned more effectively than when a conventional approach to teaching arithmetic is used. In many schools in the past it has been noted that children who approach arithmetic in the primary grades in an alert and eager manner are frustrated and bored with mathematics by the end of elementary school. Arithmetic teaching has sometimes been justly criticized as being a sterile, uninspiring matter of cajoling or forcing children into a dreary memorization of facts and routines, resulting in the children's finding difficulty in applying to problems of their everyday lives the mathematics they have supposedly learned. Perhaps the reason for the contrast between the atmosphere created by the newer programs is due to the emphasis on the discovery of ideas and relationships rather than on special practice skills. Indeed, the very process of discovery demands an intense use of previously learned skills and provides a most vital kind of practice.

One need not be disturbed by the fact that the ideas and relationships, as studied in a variety of newer programs, are expressed in terms and symbols which are new to most of us who were trained in conventional mathematics curricula, even, in many cases, through a college major. Meeting these things for the first time, one needs but to accept some modifications in the familiar vocabulary and to be willing to analyze some previously self-evident ideas in order to begin to feel at home with the new mathematics. Thus any teacher or prospective teacher, if she is willing to invest some time and effort in these things, can prepare herself to participate in the excitement and stimulation which has been generated wherever the new mathematics has been tried in the elementary school.

Teaching For Understanding

The most significant change in the new look of mathematics is that of attitude. We are no longer seeking the one best method or technique. It has been realized that there is no one way to teach, only better ways. Variety of application towards the common goal of understanding is now the trend. This emphasizes the teaching of arithmetic in its proper perspective. To teach for understanding one must take into consideration the developmental level of the children being taught in the specific classroom, and the level of difficulty of the skill or understanding being taught. Variety in presentation and application plays an important part in this concept of teaching.

As Bruner states: "Somewhere between apathy and wild excitment, there is an optimum level of aroused attention that is ideal for classroom activity. What is that level?" It is the intent of this discussion to attempt to define these levels.

If modern day education is going to educate the child to arithmetic competency and understanding, the elementary school curriculum must be built upon the following basic principles:

1. The child needs to discover, understand, and master the various mathematical concepts. For example, possession of the concept of the number 4 would be indicated by such abilities as knowing that 4 is one more than 3 and one less than 5; knowing that 4 is more than 0, 1, 2, 3, but less than 5, 6, 7, 8, 9; using symbols to represent the 4th object in a series; using number symbols to represent a set (group) such as ////, or oooo; mastering the sets of 4 in multiplication and division. In such a manner the other concepts of "4" as well as the various other number concepts would be developed.

2. The child needs to discover, understand, and master the various mathematical meanings, such as knowing that 678 is six hundreds, seven tens,

and eight ones; representing the number 678 with a pocket chart, place value frame, abacus, or counters, and explaining the role of place value in our number system and illustrating its use.

Other meanings include knowing that addition and multiplication are combining actions; whereas subtraction and division are separating actions; explaining the "equal additions" technique of compound subtraction; explaining why in subtraction of fractions, when the denominators are alike, the process is accomplished by subtracting the numerators, but not the denominators; explaining the inversion, ratio, or common denominator methods of dividing fractions; and understanding the relationship between the various processes in addition and multiplication, subtraction and division, addition and subtraction, and multiplication and division.

3. Once the child's understanding of the how and why of arithmetic develops, he needs to work for automatic mastery to become a skillful person in the use of arithmetic. This means being able to compute, check results, measure accurately, describe various numerical situations and interpret charts and graphs.

4. Throughout the entire instructional program the development of problem solving ability must be carried on. Problems give purpose to the arithmetic instruction and are basic to the introduction and application of new concepts, facts, and processes. Problem solving relates to situations in all phases of the arithmetic program. All drill and practice exercises or sequences should involve problem solving.

A mathematical problem is a challenging mathematical situation, one that requires thinking for its solution. A problem may be expressed in written form using words, or using mathematical symbols only, such as $34 + 17 = n$.

Some considerations with respect to problem solving follow:

a. Both word or verbal problems, and equations or mathematical sentences, require critical reading for their solution. For verbal problems, the child needs to visualize the situation involved. This must necessarily be part of the child's direct or vicarious experience.

b. In order to solve problems children need to recognize what is given and what is to be found, and then think out what action is needed. It is essential therefore, that all children understand clearly the mathematics of each process involved. For example, children who know subtraction only as taking away cannot be expected to recognize comparison-subtraction situations as subtraction.

c. To encourage critical reading, discussion, dramatization, etc., verbal problems should be expressed in a variety of ways. For example, extraneous numerals or other information may be included. Essential information may be omitted. All numerals may be omitted. The word "left" may be used in an addition problem; the word "together" in a subtraction problem.

d. Children should be encouraged to write several different equations for a particular verbal problem. For example: The temperature rose from 13 degrees to 64 degrees, what was the rise? Appropriate equations are:

(1) $13 + n = 64$

(2) $64 - 13 = n$

(3) $n + 13 = 64$

(4) $64 - n = 13$

e. Children should be encouraged to write several verbal problems based on a single equation.

f. Where the computation involved in solving a problem is complicated, children should be encouraged to estimate the result before computing. They should also be encouraged to use more than one algorism in computing.

Verbal problems: Verbal problems are descriptions of experience situations expressed with written words. Children interpret the words to find the mathematical problem involved.

a. Dramatizing Verbal Problems

Children interpret and/or dramatize the situation. Write equation. Children solve and rewrite as indicated. Illustrative verbal problems appropriate for this part follow:

We have 35 seashells in our collection. Donald said he would bring in 6 he has at home. How many shells will we have? (Equation: $35 + 6 = N$. Solution: $35 + 6 = 41$; We will have 41 shells.)

The boys' score is now 18. If they make 5 more points today what will their score be? (Equation: $18 + 5 = N$. Solution: $18 + 5 = 23$; Their score will be 23.)

b. Describing Experience Situations, Based on Equations

Present an appropriate equation. Children describe a probable experience situation based on the equation. Illustrations follow:

$14 + 7 = N$
I had 14 cents. My father gave me 7 cents. How much money do I have now?

$16 - N = 7$
Pamela had 16 cards. She gave some to Matt. She had 7 left. How many did she give Matt?

Mrs. Baker ordered sixteen bottles of soda for a party. Seven arrived. How many were missing?

My grandmother gave me 16 cents. When I got home I had only 7 cents of it left. How much did I spend?

c. Supplying Numbers

Children suggest appropriate quantities for situations such as the following. Then they write equations to solve.

Christy earned some money and then spent part of it. Now how much money did she have? ($25 - 7 = N$ etc.)

Matt has stamps for his album. If he gives Scott some, how many stamps will he have? ($48 - 6 = N$, etc.)

d. Recognizing Extraneous Data Given

Present descriptions such as the following:

In the pet shop John counted 18 brown puppies, 5 black puppies, and 14 kittens. How many puppies were there?

On one team 4 boys struck out, 2 ran to a base, and 3 made home-runs. On the other team 3 struck out, 5 ran to a base, and 2 made home-runs. How many home-runs were there on the two teams?

e. Completing Descriptions of Experience Situations

Children suggest ways to complete descriptions, such as:

Matt wants to buy a 23¢ book.

Pamela put $4 in the bank yesterday.

Christy sold 15 boxes of Girl Scout cookies yesterday.

Problems Expressed with Mathematical Symbols

a. Names for Numbers

Present 2 additions or subtractions. Children write other names for the number expressed.

Series A	Series B
4 + 1	6 - 1
2 + 3	8 - 3
Continue	Continue

Present several additions and/or subtractions. Children cross out any that do not belong to the series, and continue each series.

Series A	Series B
5 + 4	9 + 3
1 + 9 (x)	12 - 0
3 + 6	7 - 5 (x)
2 + 7	5 + 7
Continue	Continue

Present exercises, such as the following. Children write 10 (or more) sentences for each. (Do not limit children to additions and subtractions)

8 = (4 + 4, 24 - 16, 151 + 9 - 152, 7 fours - 5 fours, etc.)
14 =
0 =
etc.

b. Evaluating Sentences as True or False

Present sentences such as the following. For each false statement children write 5 (or more) true statements.

Addition

4 + 6 = 6 + 4 (true)
7 + 0 = 4 + 0 (false) (7 + 0 = 4 + 3 + 0, 7 - 3 + 0 = 4 + 0) etc.

Subtraction

5 = 8 - 4 (false; 5 = 8 - 3, 6 - 1 = 8 - 3, etc.)
9 - 5 = 10 - 3 (false; 9 - 5 = 10 - 6, 9 - 4 = 10 - 5, etc)

c. Number families

A Number Family consists of commutative addition facts, and an inverse subtraction fact for each, e.g.

Commutative Addition Facts	Inverse Subtraction Facts
$4 + 2 = 6$	$6 - 2 = 4$
$2 + 4 = 6$	$6 - 4 = 2$

Present an addition fact and its inverse subtraction fact. Children write the other 2 related facts.

Present an addition fact and its commutative fact. Children write the 2 inverse subtraction facts.

Present one addition or subtraction fact. Children write the 3 related facts.

Present a number. Children first write another name for the number (an addition or subtraction) and then write the 4 related facts.

Present 3 numbers, e.g. 2,5, and 7. Children write as many facts as they can using these numbers (e.g. $7 - 5 = 2$)

What is being suggested is that problem-solving be treated as an integral part of the entire arithmetic program. Problem solving is not the goal of all arithmetic. It is part of the entire instructional program.

How can this be accomplished? First of all introduce each new concept by the use of true-to-life situations. These situations must be real, childlike, and entirely possible at the age level of the child.

Secondly, a multiple approach to the solution of the problem should be advocated. It should be realized that each child learns at a different rate in a different manner depending upon his overall maturity and experiences. This means that varying amounts of time and thought are needed for different children to learn each new step or concept. Relating the new concept or process to what the child already knows is necessary. Questions that ask the child to think through or explore and find out, are suggested. In this manner, each child finds the answer to the question or exercise from the standpoint of his own understanding. If the child understands how the processes operate and why, he will obtain the correct solution to the problem. The emphasis is placed upon the child's understanding of the problem situation and the processes involved. The correct answer is still important, but understanding and meaning are more important.

The following problem is an example of the multiple approach: "Matt has 42 apples. He wants to divide them into 6 equal sets (groups). How many apples will be in each set (group)?"

The children are told to solve the problem in any manner they can. Examples of possible solutions include:

(a)

$42 \div 6 = N$, $N = 7$, $42 \div 6 = 7$

There will be 7 apples in each set (group).

(b)

$$\begin{array}{r} 7 \\ 6\,\overline{)\,42} \\ \underline{42} \\ 0 \end{array} \qquad\qquad \begin{array}{r} 7 \\ \underline{\times 6} \\ 42 \end{array}$$

There will be 7 apples in each set (group).

(c) How many 6's are there in 42?

$6N = 42$

$N = 7$

$6 \times 7 = 42$

There will be 7 apples in each set (group).

(d)

$$\begin{array}{rl} 42 & \\ \underline{-6} & (1) \\ 36 & \\ \underline{-6} & (2) \\ 30 & \\ \underline{-6} & (3) \\ 24 & \\ \underline{-6} & (4) \\ 18 & \\ \underline{-6} & (5) \\ 12 & \\ \underline{-6} & (6) \\ 6 & \\ \underline{-6} & (7) \\ 0 & \end{array}$$

There will be 7 apples in each set (group).

(e)

$$\begin{array}{rl} 6 & (1) \\ \underline{+6} & (2) \\ 12 & \\ \underline{+6} & (3) \\ 18 & \\ \underline{+6} & (4) \\ 24 & \\ \underline{+6} & (5) \\ 30 & \\ \underline{+6} & (6) \\ 36 & \\ \underline{+6} & (7) \\ 42 & \end{array}$$

There will be 7 apples in each set. (group)

(f)

ooooooo ooooooo ooooooo ooooooo ooooooo ooooooo

There will be 7 apples in each set (group).

Six alternative ways of solving the same problem have been presented. In each case the correct answer was obtained, but in each case the child was operating at a different level of competency. One could not expect a child operating at level (f) to be able to operate at level (a) overnight. The emphasis is placed upon understanding and meaning, not just the correct answer. Developing these various ways of solution is a common sense teaching method, since even the most complex of arithmetic understandings must develop from very simple and basic principles. Relating the unknown to what is known through a vast variety of experiences will result in the child reasoning out for himself and thereby gaining a greater understanding of the number system.

Once these various alternative methods have been explored and discussed, the pupil is led from his level of competency toward the more efficient ways of solving the arithmetic problem. In this way the pupil develops an insight into better ways of solving the various processes.

The many instructional devices play an important role in the overall instructional program. The number line, abacus, pocket chart, place value frame, hundred's board, ten's blocks, and arithmetic notebooks are just a few that will lend background and meaning to the development of problem solving ability.

Once the various concepts and processes have been taught, give the children proving problems. These are word problems that show the child that he understands the new concept or processes. Have them solve the problem in the generally accepted manner and then prove their findings by the use of an alternative method. In so doing, the child demonstrates that he understands the process or concept and is not just merely checking his work by a purely mechanical process.

There should also be experiences with problems containing no numbers, thereby emphasizing that thinking must be used in all solutions. Problems should be given that include extraneous information that must be sorted out. Problems lacking essential information should also be given. Here the child describes what information is needed in order to solve the problem.

The overall teaching of arithmetic places the emphasis upon mathematically meaningful teaching but ties the mathematical aspects to its basic concepts and in turn to social application. The think through approach replaces the show and tell approach that has been prevalent for so many years.

The Use of Mathematics Essentials Inventories

The measurement program at every grade level should be governed by the uses to be made of the data collected. It is, of course, assumed that the information obtained through a measurement program will be used in the instructional program. In planning the mathematics program at every grade level, teachers and others responsible for the guiding of instruction usually want some information on the general mathematical achievement of the pupils. To supply this sort of information the survey type of inventory test is most useful.

Mathematics inventories have been prepared for each grade level for use by elementary teachers as they plan and evaluate their instruction for individuals and groups within their classes. These inventories are basic diagonostic instruments and should be used accordingly. The inventory for each grade level includes sample problems and exercises on all of the major concepts and skills taught at the particular grade level.

Mathematics inventories can be used for diagnosis of instructional needs in the following ways

1. Administer the mathematics inventory for the preceding grade level during the first three weeks of school. The results of this administration can be used to determine the readiness of children within the class to proceed with instruction in the areas outlined for the year. With the transition to a modern approach to mathematics, the results of this administration can also be used to determine the new concepts or skills which must be taught or reviewed.
2. Administer inventories for different grade levels to determine the specific instructional needs for individual children or groups of children within the class. Results of these administrations can also be used in planning daily mathematics lessons for these individuals and/or groups.

3. Administer the mathematics inventory for your grade level near the close of the school year or whenever all concepts and skills for the grade level have been mastered by the students. The results of this administration will be useful as a record of achievement for each child and as a basis for review or reteaching for identified areas of need. "IN NO CASE SHOULD THE INVENTORY DETERMINE THE INSTRUCTIONAL CURRICULUM FOR A PARTICULAR GRADE LEVEL."

Each time a student completes an inventory, it should be filed in the Classroom Record Folder after it has been used for diagnosis of instructional needs. When the inventories are administered to the total class or a large group within the class, the teacher should use the suggested form to record the scores of the students. The record form provides an organized format for analyzing student and class needs. (Copies of each inventory and record form are included in this section)

Copies of mathematics inventories for all grade levels in quantity are available within each school. Request copies of inventories for specific grade levels from your building principal.

The inventory tests given near the beginning of the year are an indication of progress made in previous years. There is also need for information on the effectiveness of current instruction. To meet this need several means of gathering data can be effectively used: observation of daily work, the collecting of samples of work, short oral and written tests (in upper grades only), textbook tests, extended use of inventory tests as noted above, and several extensive teacher-made tests. Probably not more than three of the latter type are needed each year. Beginning in Grades Three or Four, diagnostic tests for a few pupils and standardized achievement tests for all can supply information needed for effective instruction.

Name________________

School______________

SCHOOL CITY OF GARY
Gary, Indiana
Elementary Department

GROUP INVENTORY OF 1ST GRADE MATHEMATICS ESSENTIALS

A

5 8 80 3 6 4 21 25

B

six __________ three __________

ten __________ nine __________

eight ________

C

84 11 48

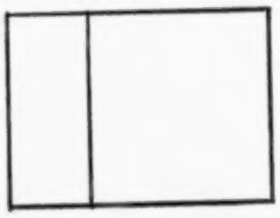
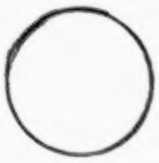
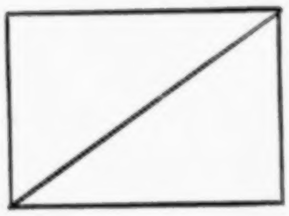
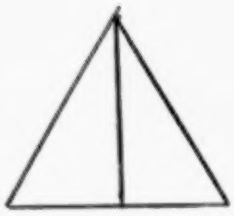
D

36 62 26

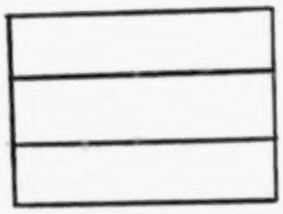
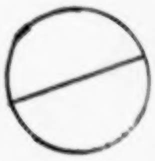
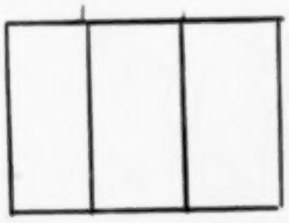
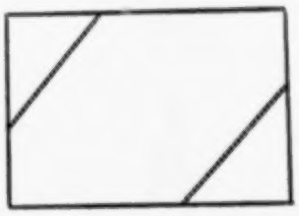
E

20 is ____ tens ____ ones or ____ + ____
57 is ____ tens ____ ones or ____ + ____
94 is ____ tens ____ ones or ____ + ____

F

G

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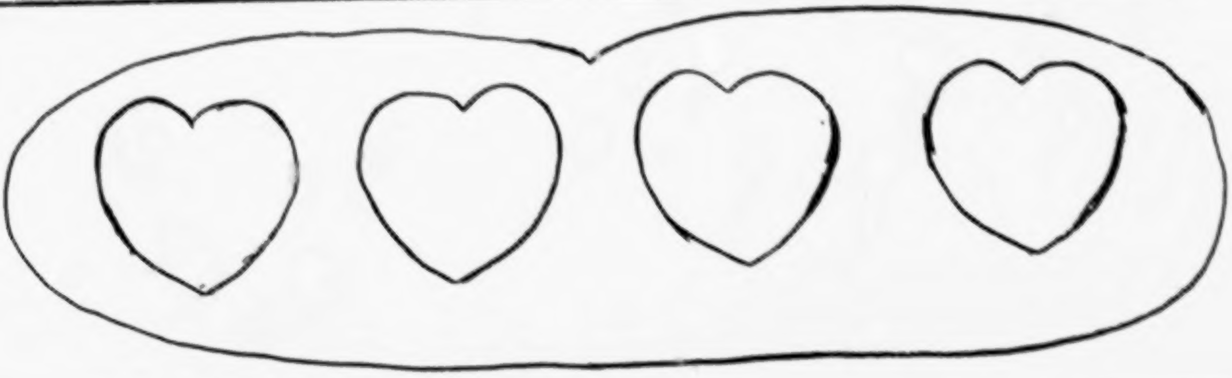


I

________ 11

________ 64

________ 80

J

28 ________

49 ________

55 ________

K

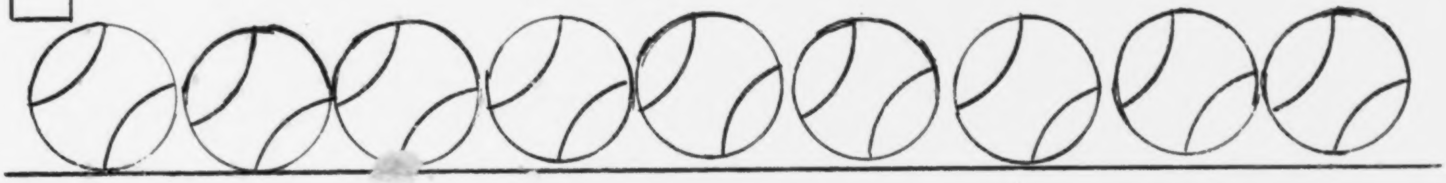
8 ________ 10

31 ________ 33

59 ________ 61

L 2, 4, 6, ___ 10, ___ 14, ___,
15, 20, 25, ___ 35, ___, ___, ___,
30, ___, 50, ___, 70

M



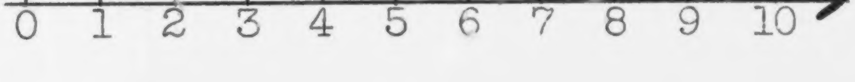
N

$\begin{array}{r} 1 \\ +6 \\ \hline \end{array}$ $\begin{array}{r} 4 \\ +6 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ +7 \\ \hline \end{array}$ $\begin{array}{r} 6 \\ +6 \\ \hline \end{array}$

O

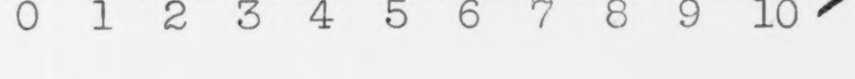
$\begin{array}{r} 7 \\ -5 \\ \hline \end{array}$ $\begin{array}{r} 6 \\ -6 \\ \hline \end{array}$ $\begin{array}{r} 10 \\ -2 \\ \hline \end{array}$ $\begin{array}{r} 12 \\ -9 \\ \hline \end{array}$

P



5+3= ☐

Q



9-4= ☐

> = <

R

10 ○ 7

18 ○ 39

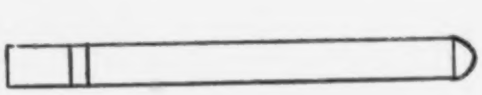
5+3 ○ 8

6-3 ○ 4

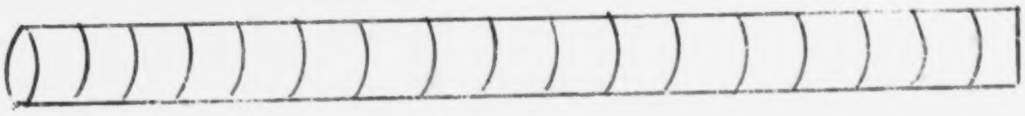
3+3 ○ 8-1

8-4 ○ 0+4

S



_______ inches

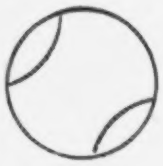


_____ inches

longer shorter

T

10¢

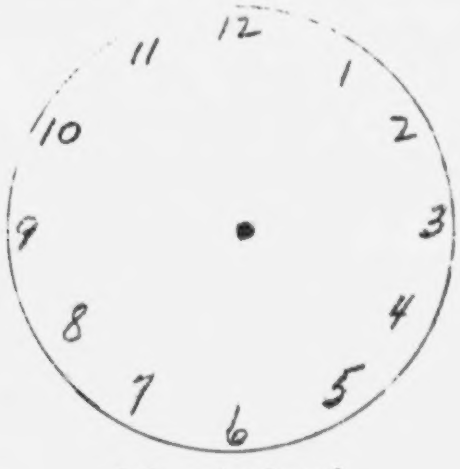


______ pennies

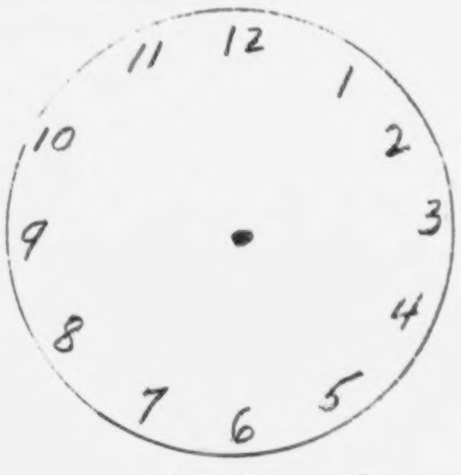
______ nickels

______ dimes

U



10 o'clock



6 o'clock

GRADE I

		Possible Number Right	Student's Score
I.	NUMBER SYSTEM		
	Number and Numerals (A-E)	13	____
	Fractions (F-H)	5	____
	Counting and Sequence (I-M)	21	____
II.	BASIC OPERATIONS		
	Addition (N)	4	____
	Subtraction (O)	4	____
III.	PROBLEM SOLVING		
	Number Line (P-Q)	2	____
	Relationships (R)	6	____
IV.	MEASUREMENT (S-U)	8	____
	TOTAL	63	

The mathematics inventory for first grade has been revised to eliminate or reduce the requirements for reading by the students in order to gain a more accurate evaluation of mathematical understanding. All instructions for the completion of this inventory are given orally by the teacher to the group. The teacher's oral instructions for the administration of this inventory follow.

RECOGNIZING NUMERALS

A "Put a cross on the numeral six."

"Put a cross on the numeral twenty-five."

"Put a cross on the numeral eighty."

NUMBER WORDS

B "Write the numeral for each number word."

PLACE VALUE

C "Draw a ring around the numeral that means 8 tens and 4 ones."

D "Draw a ring around the numeral that means 2 tens and 6 ones."

E "Write in the missing numerals."

Answers

20 is 2 tens 0 ones or 20 + 0
57 is 5 tens 7 ones or 50 + 7
94 is 9 tens 4 ones or 90 + 4

UNDERSTANDING FRACTIONS

F "Put a mark on the pictures that show halves."

G "Put a mark on the pictures that show thirds."

H "Color one-half of this set."

NUMBER SEQUENCE

I "Write the numerals that come just before these numerals."

J "Write the numerals that come just before these numerals."

K "Write the numerals that come right between these numerals."

L "Write in the missing numerals."

ORDINAL NUMBERS

M "Put a mark on the first ball."

"Put a mark on the fourth ball."

"Put a mark on the seventh ball."

ADDITION

N "Do these addition problems."

SUBTRACTION

O "Do these subtraction problems."

USE OF NUMBER LINE

P Q "Use the number line to solve these equations."

RELATIONSHIPS

R "Place a sign for greater than, less than, or equals in each circle."

MEASUREMENT

S "About how many inches long is the crayon?" (Answer - 2 inches)

"About how many inches long is the straw?" (Answer - 5 inches)

"Is the straw longer or shorter than the crayon? Draw a ring around the right word."

MONEY

T "If you bought this ball for ten cents --

How many pennies would you need to pay for it?

How many nickels would you need to pay for it?

How many dimes would you need to pay for it?"

TIME

U "Make the hands on the first clock to show 10 o'clock."

"Make the hands on the other clock to show 6 o'clock."

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 1ST GRADE MATHEMATICS ESSENTIALS

	Part	I			II		III		IV	
		A-E	F-H	I-M	N	O	P-Q	R	S-U	Total
Names of Pupils	Possible Score:	13	5	21	4	4	2	6	8	63

SCHOOL CITY OF GARY
Gary, Indiana
Elementary Department

GROUP INVENTORY OF 2ND GRADE MATHEMATICS ESSENTIALS

I. NUMBER SYSTEM

A. Recognizing Numerals

Put a cross on one hundred forty-nine, one hundred six, and forty.

32 5 40 98 106 149 19

B. Understanding Number Value

Fill in the missing numerals.

1 hundred + 2 tens + 4 ones= _______

2 hundred + 0 tens + 0 ones = _______

178 = _____hundreds + _____tens +_____ones

2071 = _____thousands + _____hundreds +
_____ tens + _____ ones

C. Write a Numeral for Each Number Word

thirteen _______

six hundred nineteen _______

four hundred thirty-two _______

eight hundred fifty _______

two hundred one _______

ninety-seven _______

D. Number Sequence

Write the number just before and just after.

_____100_____ , _____350_____ , _____499_____

E. Skip Counting by 2's, 5's, and 10's

Write the missing numerals

5 10 15 20 ____ ____ ____ ____

16 26 36 46 ____ ____ ____ 86 ____

136 138 ____ 142 ____ ____

154 152 ____ 148 ____ ____

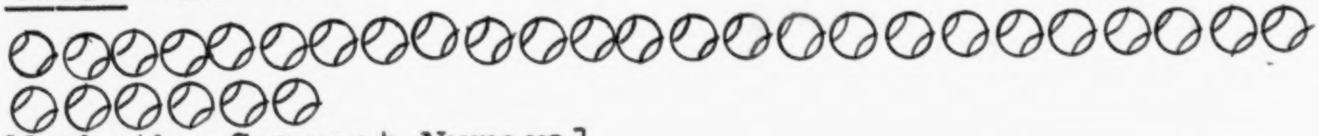
F. Knowing Odd and Even Numbers

Mark the even numbers

20 7 19 4 6 33 12 10

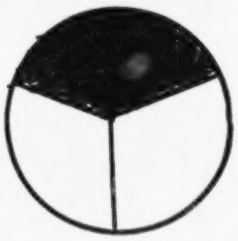
G. Ordinals

Mark the seventh, thirteenth, fifteenth, and twenty-third balls.

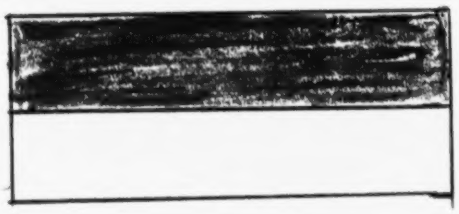


H. Mark the Correct Numeral

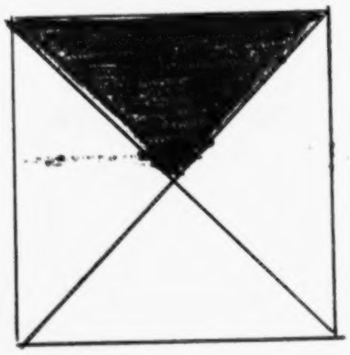
What part is shaded?



$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{3}$



$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{3}$



$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{3}$

II.BASIC OPERATIONS

A. Addition

4 + (3 + 8)= ____

(2 + 4) + 9= ____

$$\begin{array}{r} 426 \\ +\underline{351} \end{array} \qquad \begin{array}{r} 625 \\ +\underline{173} \end{array} \qquad \begin{array}{r} 221 \\ 432 \\ +\underline{\ 46} \end{array}$$

B. Subtraction

18 - 5 = ____

15 - 6 = ____

13 - ____ = 6

14 - 8 = ____

$$\begin{array}{r} 657 \\ \underline{-321} \end{array} \qquad \begin{array}{r} 794 \\ \underline{-332} \end{array} \qquad \begin{array}{r} 2,483 \\ \underline{-142} \end{array}$$

C. Multiplication

Write the missing numerals.

8 twos = _____

2 nines = _____

6 threes = _____

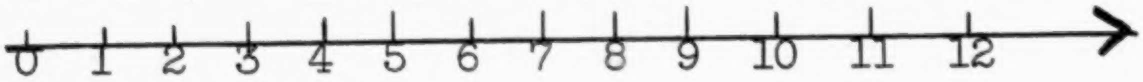
3 fives = _____

3 sixes = _____

2 sevens = _____

D. Solve by Using the Number Line

How many socks would you have if you had four pairs?



III. PROBLEM SOLVING

A. Write the Correct Signs

>, <, or = in each ○

95 - 33 ○ 60

58 - 13 ○ 68 - 23

48 - 15 ○ 30

52 + 16 ○ 69 - 7

45 + 30 ○ 70 + 5

27 + 27 ○ 50

B. Solve the Equations

(6 + 4) + 9 = ______

2 + (5 + 3) = ______

8 + 5 = 10 + ___ = _____

7 + 5 = ____ + 2 = _____

IV. MEASUREMENT

A. Money Concepts

$1.00= ________cents

$1.00= ________nickels

$1.00= ________dimes

$1.00= _______quarters

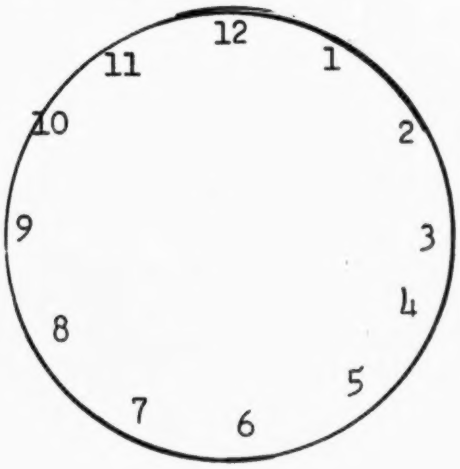
$1.00= _______half-dollars

B. Measurement

Draw a line to match the words in **A** with words in **B**.

A.	**B.**
12 inches	1 quart
4 quarts	1 hour
2 pints	1 foot
36 inches	1 yard
10 dimes	1 dollar
60 minutes	1 gallon

C. Time



Make the clock show 10:30.

GRADE 2

	Possible Number Right	Student's Score
I. NUMBER SYSTEM		
Numbers and Numerals (A-C)	13	______
Counting and Sequence (D-G)	16	______
Fractions (H)	3	______
II. BASIC OPERATIONS		
Addition (A)	5	______
Subtraction (B)	7	______
Multiplication (C-D)	7	______
III. PROBLEM SOLVING		
Relationships (A)	6	______
Equations (B)	4	______
IV. MEASUREMENT		
Money	5	______
Units of Measurement (B)	6	______
Time (C)	1	______
TOTAL	73	

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 2ND GRADE MATHEMATICS ESSENTIALS

	Part	I			II			III		IV			
		A-C	D-G	H	A	B	C-D	A	B	A	B	C	Total
Names of Pupils	Possible Scores	13	16	3	5	7	7	6	4	5	6	1	73

SCHOOL CITY OF GARY
Gary, Indiana

Elementary Department

GROUP INVENTORY OF 3RD GRADE MATHEMATICS ESSENTIALS

I. NUMBER SYSTEMS

A. Fill in the missing numerals.

7,528 = ____thousands, ____hundreds, ____tens, ____ones

8,207 = ____thousands, ____hundreds, ____tens, ____ones

614 = ____thousands, ____hundreds, ____tens, ____ones.

B. Write the numerals for:

89 tens __________

13 hundreds, 6 tens, and 4 ones __________

C. Write the Arabic numerals for:

XXIV = __________

XIX = __________

Write the Roman numerals for:

36 __________ 22 __________

D. Write fractions for:

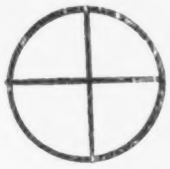
three-fourths __________

two-eighths __________

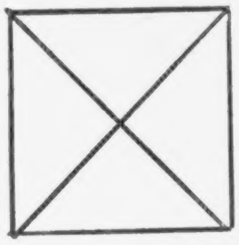
one-half __________

Shade the figures to show these fractional parts

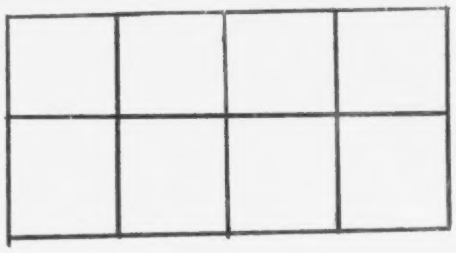
2/4



3/4



5/8



5 eighths + 2 eighths = __________eighths

3 fourths + 1 fourth = __________fourths

II. BASIC OPERATIONS

A. Addition

$$\begin{array}{rrrr} 25 & 416 & \$2.39 & 144 \\ 31 & 350 & +\ \ 6.43 & +\ 275 \\ +\ 42 & +\ 122 & \hline & \hline \\ \hline \end{array}$$

$$\begin{array}{rrrr} 286 & 439 & 335 & \$4.47 \\ +\ 697 & +\ 144 & +\ 582 & +\ 3.75 \\ \hline \end{array}$$

B. Subtraction

$$\begin{array}{rrrrrr} 689 & 658 & \$5.16 & 429 & \$5.53 & 931 \\ -\ 68 & -\ 428 & -\ 3.75 & -\ 154 & -\ 2.66 & -736 \\ \hline \end{array}$$

C. Multiplication

$$\begin{array}{rrrrrr} 312 & 212 & 320 & 545 & 634 & 475 \\ \times\ 3 & \times\ 4 & \times\ 6 & \times\ 5 & \times\ 3 & \times\ 6 \\ \hline \end{array}$$

D. Division

$5\overline{)35}$ $\quad 2\overline{)24}$ $\quad 4\overline{)320}$ $\quad 2\overline{)156}$ $\quad 3\overline{)438}$ $\quad 6\overline{)498}$

III. PROBLEM SOLVING

A. Solve the equations

(5 x 0) + (0 x 5) = ________ (5 x 3) + 4 = ________

4 + (3 + 0) = ________ (3 x 10)- 8= ________

B. Relationships

Use >, < , or = in the ○

3 x 4 ○ 2 x 7 3 x 9 ○ 4 x 7

24 ÷ 4 ○ 36 ÷ 6 64 ○ 6 tens and 4 ones

5 x 6 ○ 7 x 4

C. Word Problems

1. Tom wrote 45 words, 168 words, and 107 words. Altogether he wrote how many words? ________

2. Ann has \$3.05. Jane has \$2.65. How much money has Ann? ________

3. Mother needs 3 pounds of butter. There are 4 sticks in each pound. How many sticks of butter does she need? ________

4. There are 36 inches in 1 yard. How many inches are there in $\frac{1}{4}$ yard?_____

IV. MEASUREMENT AND GEOMETRY

A. Write true or false after each

3 quarts = $\frac{1}{2}$ gallon ________

4 inches = $\frac{1}{3}$ foot ________

40 seconds = $\frac{1}{2}$ minute ________

6 months = $\frac{1}{2}$ year ________

1 foot = $\frac{1}{3}$ yard ________

5 ounces = $\frac{1}{4}$ pound ________

1 pint = $\frac{1}{2}$ quart ________

10 things = $\frac{1}{2}$ dozen ________

B. Money

Fill in the missing numerals

1 dollar = ________ pennies + 6 nickels

1 dollar = 5 dimes + ________ quarters

1 dollar = ________ half-dollars

4 dimes = 2 nickels + ________ pennies

3 nickels = 1 dime + ________ pennies

C. Geometry

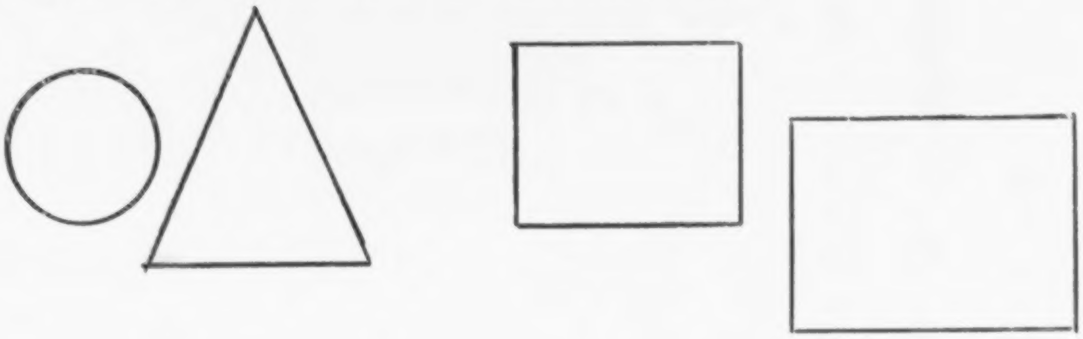
Draw lines to match the figures with the correct names.

Rectangle

Square

Circle

Triangle



GRADE 3

	Possible Number Right	Student's Score
I. NUMBER SYSTEM		
Numbers and Numerals (A-B)	5	
Roman Numerals (C)	4	
Fractional Numbers (D)	8	
II. BASIC OPERATIONS		
Addition (A)	8	
Subtraction (B)	6	
Multiplication (C)	6	
Division (D)	6	
III. PROBLEM SOLVING		
Equations (A)	4	
Relationships (B)	5	
Word Problems (C)	4	
IV. MEASUREMENT AND GEOMETRY		
Measurement (A)	8	
Money (B)	5	
Geometry (C)	4	
TOTAL	73	

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 3RD GRADE MATHEMATICS ESSENTIALS

	Part	I			II				III			IV			
		A-B	C	D	A	B	C	D	A	B	C	A	B	C	Total
Names of Pupils	Possible Score:	5	4	8	8	5	6	6	4	5	4	8	5	4	73

SCHOOL CITY OF GARY
Gary, Indiana
Elementary Department

GROUP INVENTORY OF 4TH GRADE MATHEMATICS ESSENTIALS

I. UNDERSTANDING OUR NUMBER SYSTEM

A. Write a numeral for each of the following:

1. Seventy thousand, fifty ____________________

2. Ninety four million, two hundred forty-four ____________________

3. 19 hundreds, 11 ones ____________________

4. 3 ten thousands, 5 thousands ____________________

B. In the Numeral 236,149,850

236 tells us how many __________

149 tells us how many __________

→ 9 means 9 __________

→ 4 means 4 __________

→ 1 means 1 __________

C. Understanding Roman Numerals

1. Write Arabic numerals for these: XXIV _________ CXIX _________

2. Write Roman numerals for these: 96 _________ 140 _________

D. Fractions, Decimals, Percents

1. {○○△△△ □□□}

What part of this set are the triangles? _______________

What part of this set are the squares? _______________

What part of this set are the circles? _______________

2. What is the lowest term fraction that can be used to name the following:

$\frac{9}{12}$ .75 $\frac{12}{16}$ 75% _______________

II. BASIC OPERATIONS

A. Addition

$$\begin{array}{r}6214\\3046\\4697\\+\ 9305\\\hline\end{array}\qquad\begin{array}{r}38\\63\\41\\59\\+\ 72\\\hline\end{array}$$

$18 + 309\ \square = 487$

$2.43 + 0.35 = \square$

$\frac{1}{2} + \frac{3}{8} = \square$

$\frac{3}{4} + \frac{2}{4} = \square$

B. Subtraction

$$\begin{array}{r}3145\\-\ 1057\\\hline\end{array}\qquad\begin{array}{r}\$30.00\\-\ 8.94\\\hline\end{array}$$

$(741 - 36) + (27 - 8) = \square$

$\frac{7}{8} - \frac{4}{8} = \square$

$\frac{7}{10} - \frac{1}{10} = \square$

C. Multiplication

$72 = \square \times 9$

$48 = 6 \times \square$

$$\begin{array}{r}6325\\\times\ \ 7\\\hline\end{array}\qquad\begin{array}{r}49\\\times\ 26\\\hline\end{array}$$

D. Division

$4\,\overline{)\,3564}$

$7\,\overline{)\,512}$

$31\,\overline{)\,279}$

$23\,\overline{)\,156}$

III. PROBLEM SOLVING

A. Make each statement true by completing the box with $>$, $<$, cr =.

$\frac{1}{2}$ ☐ .5 $\frac{1}{4}$ ☐ $\frac{3}{8}$ 1 ☐ $\frac{16}{16}$

$\frac{5}{4}$ ☐ $1\frac{1}{2}$ $\frac{12}{8}$ ☐ $1 + \frac{1}{2}$ .7 ☐ .55

$\frac{6}{10}$ ☐ .06   .01 ☐ 1% ☐ $\frac{1}{100}$

B. Word Problems

1. Father took Jim to the ball game. He spent \$2.50 for tickets, 20¢ for peanuts, 80¢ for hot dogs. How much did he spend?__________

2. There are 32 children going on a trip. The cost is 10¢ per child. $\frac{1}{4}$ of the children have not paid. How much money has been paid?_____

3. How many feet of fence will it take to enclose a square lot which is $25\frac{1}{2}$ feet on each side?__________

4. Mrs. Wilson bought 9 lbs. of hamburger meat for the school cafeteria. She plans to use 6 oz. in each hamburger. How many hamburgers can she make?__________

IV. MEASUREMENT AND GEOMETRY

A. Complete the following:

$\frac{3}{4}$ of an hour = __________minutes

2 ft. 2 in. is __________inches

38 in. is __________yds.__________inches

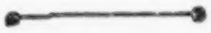
$1\frac{1}{4}$ lbs. is __________ounces

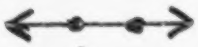
$\frac{1}{4}$ gallon is __________quart

65 pennies=__________nickels

68 cents = __________dimes,__________nickels, __________pennies

B. What does each of these drawings picture?


a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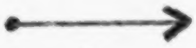
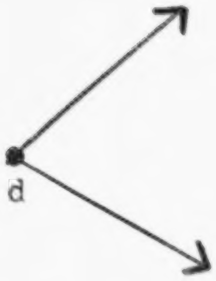

c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a________ b________ c________ d________

C. Draw a rectangle. Mark a point on the figure and label it point "a."

Mark a point inside the figure and label it point "b."

Mark a point outside the figure and label it point "c."

GRADE 4

MATHEMATICS INVENTORY ANSWER SHEET

I. UNDERSTANDING NUMBER SYSTEMS

A.

1. 70,050
2. 94,000,244
3. 1,911
4. 35,000

B. 236 tells us how many millions
149 tells us how many thousands
→ 9 means 9 thousands
→ 4 means 4 ten thousands (40 thousands)
→ 1 means 1 hundred thousand

C.

1. 24 119
2. XCVI CXL

D.

1. $\frac{3}{8}$; $\frac{3}{8}$; $\frac{2}{8}$

2. $\frac{3}{4}$

II. BASIC OPERATIONS

A.

$$\begin{array}{r} 6214 \\ 3046 \\ 4697 \\ 9305 \\ \hline 23262 \end{array} \qquad \begin{array}{r} 38 \\ 63 \\ 41 \\ 59 \\ 72 \\ \hline 273 \end{array}$$

$18 + 309 + \boxed{160} = 487$

$2.43 + .35 = \boxed{2.78}$

$\frac{3}{4} + \frac{2}{4} = \boxed{\frac{5}{4}}$ or $\boxed{1\frac{1}{4}}$ $\qquad \frac{1}{2} + \frac{3}{8} = \boxed{\frac{7}{8}}$

B.

$$\begin{array}{r} 3145 \\ -1057 \\ \hline 2088 \end{array} \qquad \begin{array}{r} \$30.00 \\ -\ 8.94 \\ \hline \$21.06 \end{array}$$

$(741-36) + (27-8) = \boxed{724}$

$\frac{7}{8} - \frac{4}{8} = \boxed{\frac{3}{8}} \qquad \frac{7}{10} - \frac{1}{10} = \boxed{\frac{6}{10}} = \boxed{\frac{3}{5}}$

C.

$72 = \boxed{8} \times 9$

$48 = 6 \times \boxed{8}$

$$\begin{array}{r} 6325 \\ \times\ 7 \\ \hline 44275 \end{array} \qquad \begin{array}{r} 49 \\ \times\ 26 \\ \hline 1274 \end{array}$$

D.

$$\begin{array}{r} 891 \\ 4\,\overline{)3564} \\ \underline{3200} \\ 364 \\ \underline{360} \\ 4 \\ \underline{4} \end{array} \qquad \begin{array}{r} 73 \\ 7\,\overline{)512} \\ \underline{490} \\ 22 \\ \underline{21} \\ 1 \end{array} \qquad \begin{array}{r} 9 \\ 31\,\overline{)279} \\ \underline{279} \end{array} \qquad \begin{array}{r} 6 \\ 23\,\overline{)156} \\ \underline{138} \\ 18 \end{array}$$

III. PROBLEM SOLVING

A. Relationships

$\frac{1}{2} \boxed{=} .5$ $\frac{1}{4} \boxed{<} \frac{3}{8}$ $1 \boxed{=} \frac{16}{16}$ $\frac{5}{4} \boxed{<} 1\frac{1}{2}$

$.7 \boxed{>} .55$ $\frac{6}{10} \boxed{>} .06$ $\frac{12}{8} \boxed{=} 1 + \frac{1}{2}$ $.01 \boxed{=} 1\% \boxed{=} \frac{1}{100}$

B. Word Problems

1. $3.50
2. $2.40
3. 102 feet
4. 24

IV. MEASUREMENT AND GEOMETRY

A. $\frac{3}{4}$ of an hour = 45 minutes

2 ft. 2 in. is 26 inches

38 in. is 1 yd. 2 in.

$1\frac{1}{4}$ lbs. is 20 oz.

$\frac{1}{4}$ gallon is 1 quart

65 pennies = 13 nickels

68 cents = 6 dimes, 1 nickel, 3 pennies (Any combinations that = 68 cents are acceptable.)

B. (a) line segment (b) line (c) ray (d) angle

C. a. $\boxed{\text{b.}}$ c.

44

GRADE 4

		Possible Number Right	Student's Score
I.	NUMBER SYSTEM		
	Numbers and Numerals (A-B)	9	___
	Roman Numerals (C)	4	___
	Fractions (D)	4	___
II.	BASIC OPERATIONS		
	Addition (A)	6	___
	Subtraction (B)	5	___
	Multiplication (C)	4	___
	Division (D)	4	___
III.	PROBLEM SOLVING		
	Relationships (A)	8	___
	Word Problems (B)	4	___
IV.	MEASUREMENT AND GEOMETRY		
	Measurement (A)	7	___
	Geometry (B-C)	7	___
	TOTAL	62	

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 4TH GRADE MATHEMATICS ESSENTIALS

Part	I			II				III		IV		
	A-B	C	D	A	B	C	D	A	B	A	B-C	Total
Possible Score:	9	4	4	6	5	4	4	8	4	7	7	62
Name of Pupils												

SCHOOL CITY OF GARY
Gary, Indiana

Elementary Department

GROUP INVENTORY OF 5TH GRADE MATHEMATICS ESSENTIALS

I. UNDERSTANDING OUR NUMBER SYSTEMS

A. In the number 1,007,562,403

one means __________

seven means_________

six means___________

B. Write the numerals for eight million, sixty-two thousand, five hundred three in expanded notation.

(8 x 1,000,000) + (6 x _____) + (_____ x 1000) + (_____ x _____) +(3 x_____)

C. Circle the largest fraction

$\frac{5}{8}$ $\frac{9}{16}$ $\frac{3}{4}$ $\frac{1}{2}$

D. Understanding Fractions, Decimals, and Percent

Rewrite: $\frac{9}{10}$ as a decimal fraction __________

2.3 as a mixed number ______________

1 $\frac{7}{100}$ as a decimal fraction________

0.40 as a percent___________________

0.57 as a common fraction__________

2% as a decimal fraction___________

E. Circle the smallest decimal fraction in each row.

.5 .05 .005 .0005

.5 .55 .555 .5555

F. Understanding Roman Numerals

In Arabic numerals

CXXIX means ________

XLVIII means ________

In Roman numerals

389 is ____________

165 is ____________

II. BASIC OPERATIONS

A. Addition

$$\begin{array}{r} 539 \\ 453 \\ 697 \\ 460 \\ +\ 259 \\ \hline \end{array} \qquad \begin{array}{r} \$12.06 \\ .35 \\ 4.20 \\ 9.98 \\ +\ .06 \\ \hline \end{array} \qquad \begin{array}{r} 9\frac{5}{6} \\ +\ 3\frac{2}{6} \\ \hline \end{array} \qquad \begin{array}{r} 5\frac{3}{4} \\ +\ 3\frac{5}{8} \\ \hline \end{array} \qquad \frac{3}{8} + \square = 1$$

B. Subtraction

$$\begin{array}{r} 7084 \\ -\ 3956 \\ \hline \end{array} \qquad \begin{array}{r} \$38.03 \\ -\ 29.07 \\ \hline \end{array} \qquad \begin{array}{r} 6\frac{5}{8} \\ -\ 3\frac{1}{4} \\ \hline \end{array} \qquad \begin{array}{r} 8\frac{3}{4} \\ -\ 4\frac{7}{8} \\ \hline \end{array}$$

C. Multiplication

$$\begin{array}{r} 353 \\ \times 102 \\ \hline \end{array} \qquad \begin{array}{r} 8.1 \\ \times\ 5.6 \\ \hline \end{array} \qquad \frac{4}{5} \times \frac{2}{3} = \square \qquad 2\frac{1}{5} \times 3\frac{1}{3} = \square$$

D. Division

$$23\,\overline{)\,6948} \qquad 45\,\overline{)\,27.90} \qquad 9 \div \frac{2}{3} = \square$$

$$6\frac{1}{2} \div 3\frac{3}{10} = \square$$

III. PROBLEM SOLVING

A. Use the following symbols to make each problem true. $\boxed{> \quad < \quad =}$

$(1126 \div 2)$ ________ 563

$\frac{6}{8}$ ________ $\frac{3}{4}$

(5% of 60) ________ 12

$1\frac{1}{3}$ ________ $\frac{4}{3}$

$\frac{2}{10}$ ________ 20%

B. Solve these problems for N.

$2 = \frac{N}{4}$ N = ________ $\frac{2}{3} = \frac{N}{9}$ N = ________

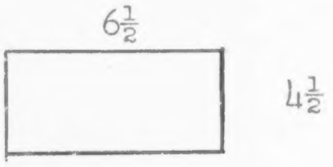
C. Word Problems

1. Tom saved \$1.25 one week, \$.85 the next, \$1.10 the next, and \$1.00 the next. What is the average amount he saved in one week?_______

2. Bob has a square tennis court. How much fencing would he have to buy to surround the court if one side measures 50 feet?__________

3. Our school enrollment is 760. Ten percent of the students are on the honor roll. How many children are on the honor roll?________

4. Mary bought some cloth to make a dress. The cloth measured $4\frac{1}{2}$ yards by $6\frac{1}{2}$ yards. How much did the cloth cost at \$1.00 a square yard?__________



5. Bob saved 9 quarters a month in his bank. How much money did he save in one year?__________

IV. MEASUREMENT AND GEOMETRY

A. Complete the following;

1 square yard = ________ square feet 4 yards = _______ feet

B. Write the number of the geometric form that matches with its name in the left-hand column.

_____ Radius

_____ Parallel Lines

_____ Right Angle

_____ Plane

_____ Line Segment

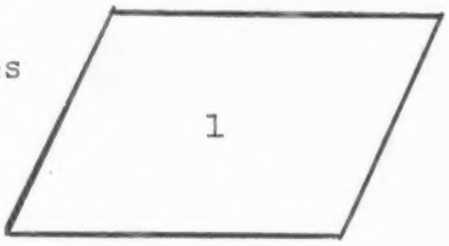
_____ Triangle

_____ Cylinder

_____ Cone

_____ Sphere

_____ Open figure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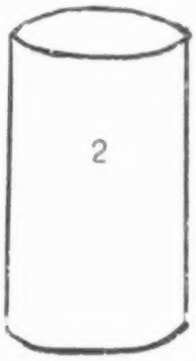
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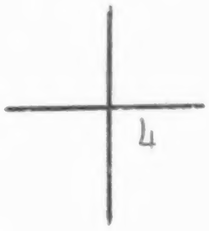
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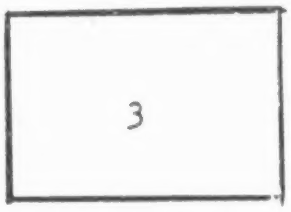
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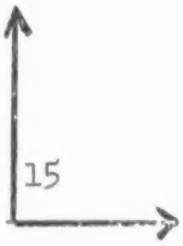
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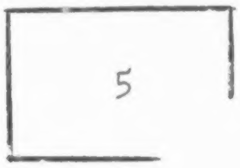
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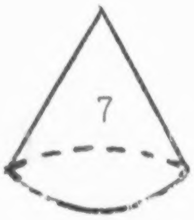
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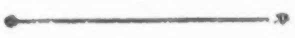
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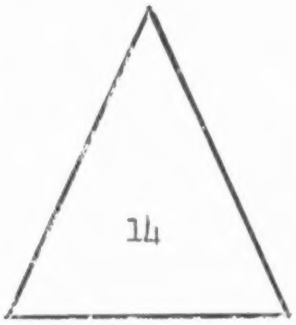
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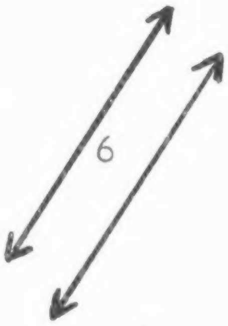
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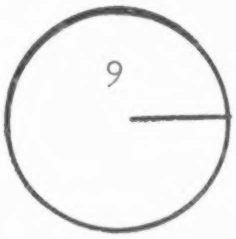
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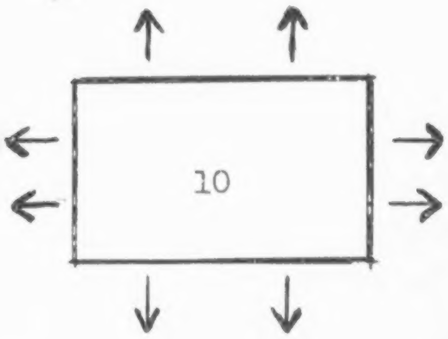
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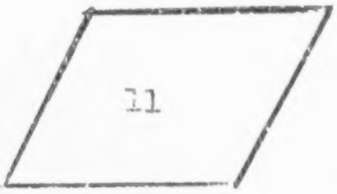
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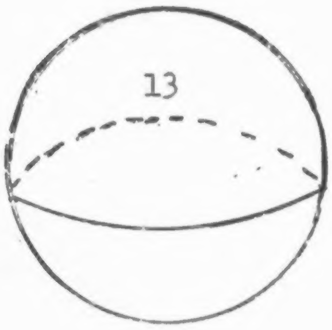
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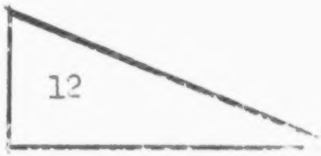
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GRADE 5

	Possible Number Right	Student's Score
I. NUMBER SYSTEM		
Numbers and Numerals (A-E)	13	____
Roman Numerals (F)	4	____
II. BASIC OPERATIONS		
Addition (A)	5	____
Subtraction (B)	4	____
Multiplication (C)	4	____
Division (D)	4	____
III. PROBLEM SOLVING		
Relationships (A)	5	____
Equations (B)	2	____
Word Problems (C)	5	____
IV. MEASUREMENT AND GEOMETRY		
Measurement (A)	2	____
Geometry (B)	10	____
TOTAL	58	

GRADE 5

MATHEMATICS INVENTORY ANSWER SHEET

I. UNDERSTANDING OUR NUMBER SYSTEM

A. In the number 1,007,562,403

one means 1 billion
seven means 7 millions
six means 6 ten thousands or 60 thousands

B. (8 x 1,000,000) + (6 x 10,000) + (2 x 1,000) +(5 x 100) + (3 x 1)

C. $\frac{3}{4}$

D. $\frac{9}{10} = .9$ $\quad 1\frac{7}{100} = 1.07$ $\quad .57 = \frac{57}{100}$

$2.3 = 2\frac{3}{10}$ $\quad .40 = 40\%$ $\quad 2\% = .02$

E. Circle the smallest fractions

(1) [✓ .0005] (2) [✓ .5]

F. Roman Numerals

CXXIX 129
XLVIII 48
389 is CCCLXXXIX
165 is CLXV

II. BASIC OPERATIONS

(Check Test)

III. PROBLEM SOLVING

A. (1) = (2) = (3) $<$ (4) = (5) =

B. (1) $n = 8$ (2) $n = 6$

C. (1) $1.05 (2) 200 ft. (3) 76 (4) $29.25 (5) $27.00

IV. MEASUREMENT AND GEOMETRY

A. 1 sq. yd. = 9 sq. ft. 4 yds. = 12 ft.

B. (Check test)

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 5TH GRADE MATHEMATICS ESSENTIALS

	Part	I		II				III			IV		
		A-E	F	A	B	C	D	A	B	C	A	B	Total
Names of Pupils	Possible Score	[illegible]	4	3	[illegible]	4	4	5	2	[illegible]	2	10	58

SCHOOL CITY OF GARY
Gary, Indiana
Elementary Department

GROUP INVENTORY OF 6TH GRADE MATHEMATICS ESSENTIALS

I. UNDERSTANDING OUR NUMBER SYSTEM

A. Write another numeral for each of the following:

Two billion, three hundred thousand______________

Three million, two hundred and nine thousandths______________

$(1 \times 10^2) + (2 \times 10^1) + (6 \times \frac{1}{10^2}) =$ ______________

B. Expand each number using exponents

24,092=______________

945.19=______________

C. 6,250 is equal to how many tens?______________

D. 72,812 is equal to how many hundreds?______________

E. Rename the ratio 4 to 5 as a fraction_________, decimal_________, percent _________.

F. Indicate whether each statement is TRUE or FALSE.

______1. The product of two odd numbers is always odd.

______2. Dividing by 4 is the same as multiplying by $\frac{1}{4}$.

______3. Multiplying .96 by $\frac{100}{100}$ changes the number.

______4. Inverse operations are useful in checking subtraction and division.

G. The following patterns illustrate certain basic properties. Identify the properties.

$\boxed{36} + \triangle 12 = \triangle 12 + \boxed{36}$ ______________

$(\boxed{2} \times \triangle 3) + (\boxed{2} \times \triangledown 4) = \boxed{2}(\triangle 3 + \triangledown 4)$ ______________

$\square \times (\triangle \times \triangledown) = (\square \times \triangle) \times \triangledown$ ______________

$\square \times \triangle = \square$ ______________

$\frac{\square}{\triangle} \times \frac{\triangle}{\square} = 1$ ______________

II. BASIC OPERATIONS

-- Perform the indicated operations.

$4.8 + 23.53 + 9.76 + 17.8 + 4.98$

$$\begin{array}{r} 5\frac{4}{5} \\ 7\frac{3}{8} \\ +\ 1\frac{1}{10} \\ \hline \end{array} \qquad \begin{array}{r} 39\frac{7}{8} \\ -\ 11\frac{5}{6} \\ \hline \end{array} \qquad \begin{array}{r} 9 \text{ lb. } 8\frac{1}{2} \text{ oz.} \\ -\ 6 \text{ lb. } 6\frac{3}{4} \text{ oz.} \\ \hline \end{array}$$

$$7\frac{1}{2} \times 3\frac{1}{3} \qquad 9 \times 8\frac{1}{2} \qquad \begin{array}{r} 3.25 \\ \times\ 1.2 \\ \hline \end{array}$$

$$36 \div \frac{4}{9} \qquad 3\frac{1}{4} \div 4\frac{2}{3} \qquad \frac{624}{2.5}$$

$$223\,\overline{\smash{)}43789} \qquad \begin{array}{r} 609 \\ \times\ 507 \\ \hline \end{array} \qquad \begin{array}{r} 10{,}302 \\ -\ 9{,}305 \\ \hline \end{array}$$

III. PROBLEM SOLVING

A. Relationships

Make each sentence true by replacing the ☐ with $>$, $<$, or $=$.

$\frac{30}{60}$ ☐ 50% $\qquad \frac{7}{5}$ ☐ .14 $\qquad \frac{1}{10}$ ☐ 0.1

3^3 ☐ 5^2 $\qquad \frac{1}{100}$ ☐ $\frac{1}{10^2}$

B. Solving Equations

-- Solve the following equations for "n".

$\frac{n}{6} = \frac{10}{15}$ $n \times 72 = 66093$ $89.43 \times n = 84.93$

$6n = 42$ $n = 1.4 \times (49 - 28)$

C. Word Problems

1. Jane paid 45¢ for $\frac{3}{4}$ of a yard of cloth. How much was it per yard?____

2. The temperature rose from 10.2 degrees below zero to 3.5 degrees above freezing. How much did it rise?__________

3. Bob spelled 5 words incorrectly on a test of 20 words. What fractional part did he spell correctly? _______ What percent?_______

4. If 72 is 60% of a number, what is the number?_______

5. Find the area of a triangle whose base is 6 ft. and altitude is 11 ft._______

6. Joan has $250 in her savings account. How much interest will her money earn in one year at 4%?_______

7. A parallelogram has a perimeter of 40 ft. Find the length of each side if one side is 7 ft._______

8. Find the circumference of a circle whose radius is 8 ft._______

IV. GEOMETRY

-- Matching

Place the letter of the definition in the right-hand column in the space before the proper geometric form in the left-hand column.

________ angle

________ congruent figures

________ equilateral triangle

________ trapezoid

________ line

________ circle

________ line segment

________ space figures

a. A set of points in a plane, a given distance from its center

b. A three-sided closed plane figure having all sides equal

c. A never-ending set of points (having no end points)

d. Geometric figures that lie in more than one plane

e. Figures having the same shape and size

f. The set of points in two rays with a common end point

g. A quadrilateral with only two of its sides parallel

h. A set of points having two end points

GRADE 6

		Possible Number Right	Student's Score
I.	NUMBER SYSTEM		
	Numbers and Numerals (A-E)	8	
	Basic Concepts (F-G)	9	
II.	BASIC OPERATIONS	13	
III.	PROBLEM SOLVING		
	Relationships (A)	5	
	Equations (B)	5	
	Word Problems (C)	8	
IV.	GEOMETRY	8	
	TOTAL	56	

GRADE 6

MATHEMATICS INVENTORY ANSWER SHEET

I. UNDERSTANDING OUR NUMBER SYSTEM

A. 2,000,300,000
3,000,200.009
120.06

B. $(2 \times 10^4) + (4 \times 10^3) + (9 \times 10^1) + (2 \times 10^0)$ or 2

$(9 \times 10^2) + (4 \times 10^1) + (5 \times 10^1)$ or 5

C. 625

D. 728

E. $\frac{4}{5}$ .8 80%

F. 1. True
2. True
3. False
4. True

G. Commutative
Distributive
Associative
Identity for Multiplication or Multiplicative Identity
Reciprical

II. BASIC OPERATIONS

60.87 $13\frac{51}{40}$ or $14\frac{11}{40}$ $28\frac{1}{12}$ 3 lb. $1\frac{3}{4}$ oz. 25 $76\frac{1}{2}$

3.900 81 $\frac{39}{56}$ 249.6 196 r 18 308,763 997

III. PROBLEM SOLVING

A. $\frac{30}{60} \boxed{=} 50\%$ $\frac{7}{5} \boxed{>} .14$ $\frac{1}{10^2} \boxed{<} 0.1$ $3^3 \boxed{>} 5^2$

$\frac{1}{100} \boxed{=} \frac{1}{10^2}$

B. $n = 4$ $n = 917\frac{23}{24}$ $n = 4.5$ $n = 7$ $n = 29.4$

C.

1. \$.60
2. 45.7° F (13.8° C)
3. $\frac{3}{4}$ 75%
4. 120
5. 33 sq. ft.
6. \$10.00
7. Two sides measure 7 ft. each. Other two sides measure 13 ft. each.
8. 50 $\frac{2}{7}$ ft.

IV. GEOMETRY

f	angle
e	congruent figures
b	equilateral triangle
g	trapezoid
c	line
a	circle
h	line segment
d	space figures

CLASS RECORD OF SAMPLE GROUP INVENTORY OF 6TH GRADE MATHEMATICS ESSENTIALS

	Part	I		II	III			IV	
		A-E	F-G		A	B	C		Total
Names of Pupils	Possible Score	8	9	13	5	5	8	8	56

CONCEPTS AND CONTENT OF MATHEMATICAL INSTRUCTIONAL PROGRAM

Children at all grade levels of the elementary school throughout the United States present a wide range of number abilities and interests. In a typical class one may find pupils who have a sizable amount of arithmetic knowledge and a readiness for new work. Others, in the same class, have a vague, inaccurate arithmetic background, a lack of ability to make normal progress, and frequently living in an enviornment offering a limited range of mathematical and social living experiences. The problem of improving teaching and learning arithmetic in these situations remains urgent.

Today children no longer merely try to memorize abstract symbols; they are learning at a very early age the meaning of number by making practical use of it at their home and school situations. They are learning to think quantitatively. Arithmetic has come to have such an important place in our modern life that children need to understand it as a means of effective living. To know why and when is as necessary as to know how to preform an operation in arithmetic.

The teacher's task is to direct the pupils to study numbers so that they may develop, enlarge, and clarify their ideas of the number system. Classroom procedure selected should instruct the pupils in the ways to do number thinking, and direct them into an independent study of numbers. This represents an important challenge to teachers who aim to offer guidance in the methods of studying numbers systematically.

One of the most important realizations resulting from the study of arithmetic through these past years is that thinking in arithmetic and an understanding of the mathematical meaning of numbers and processes must take precedence over the mechanical manipulation of numbers. Mathematical concepts and meanings should be developed over a long period of time through the use of socially significant situations before computational aspects are stressed.

The meaning theory of arithmetic conceives of arithmetic as an organized series of related ideas, processes and principles. It is quite the opposite of the drill theory which holds that arithmetic is a mass of separate and unrelated fragments.

Arithmetic cannot be taught as an isolated subject. All school and community living has potentialities for contributing to mathematical understanding and for further mathematical thinking.

Thinking with numbers is abstract thinking. But the young child's thinking is literal and on a concrete level. He needs a great deal of help to understand the abstractions involved in the study of number. Plan activities that will guide the child in recognizing the numerical aspects in familiar situations, introducing a variety of objective material suitable for developing number concepts and assisting the individual to make this transition from concrete thinking to abstract thinking. Gradually he will develop from simple abstractions to more complicated abstractions.

Growth at times is difficult to measure. This transition from the concrete to the abstract is important at every point and at every grade level where a new mathematical concept or process is to be reinforced. Furthermore, when a child fails to recall a combination or method of a mathematical process it is necessary to re-introduce significant situations and objective materials so that a meaningful basis is secured.

Arithmetic is more than a set of specific skills and facts. It is a system of quantitative and qualitative thinking including facts, concepts, principles, and processes which are so closely interrelated that they cannot be separated in practice. This kind of thinking develops slowly and needs the incentive of being functional. Arthmetic needs to be more of a challenge to the child's intelligence than to his memory.

It is the major goal of this guide to help teachers of arithmetic to teach in a more meaningful manner, both mathematically and socially. Instruction in such a program should:

1. Develop an appreciation of how numbers have facilitated human progress, and of the social signficiance of arithmetic in the affairs of life.
2. Develop concepts and vocabulary basic to quantitative thinking.
3. Develop understanding, accuracy, and mastery of the essential skills.
4. Develop the right attitudes and interests to form a background for continued interest in and use of mathematics.
5. Develop an inquiring attitude of mind through the use of problems for which the pupils seek pertinent facts and numbers in the content fields, in basic texts, and reference materials.

Based upon this philosophy this section is organized around the following content areas:

1. Numbers and Numerals
2. Sets and Sentences
3. Whole Numbers
4. Rational Numbers
5. Measurement & Money
6. Geometry

7. Other Systems and Bases

8. Problem Solving

Each of the areas is further developed by the inclusion of the mathematical concepts necessary for understanding and meaning, presented in ascending order of difficulty at the appropriate grade level designations.

SCOPE AND SEQUENCE OF MATHEMATICAL SKILLS AND UNDERSTANDINGS

	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6
I. Systems of Notation and Numeration						
A. Base 10-Hindu-Arabic System						
1. Things and Names of Things	X	X	X	X	X	X
2. Enumeration, Reading and Writing Numerals and Number Words	X	X	X	X	X	X
3. Place Value	X	X	X	X	X	X
4. Different Names For The Same Number	X	X	X	X	X	X
B. Other Historical Systems			X	X	X	X
C. Other Bases					X	X
II. Sets and Sentences						
A. Basic Concepts	X	X	X	X	X	X
B. Notation	X	X	X	X	X	X
III. Number Systems						
A. Whole Numbers						
1. Basic Concepts	X	X	X	X	X	X
2. Properties and Relationships	X	X	X	X	X	X
3. Operations	X	X	X	X	X	X
B. Rational Numbers						
1. Basic Concepts	X	X	X	X	X	X
2. Properties and Relationships				X	X	X
3. Operations		X	X	X	X	X
C. Integers						
1. Basic Concepts						X
2. Properties and Relationships						X
3. Operations						X
IV. Measurement, Number Line, Graphing						
A. Measurement						
1. Concepts	X	X	X	X	X	X
2. Applications	X	X	X	X	X	X
B. Number Line	X	X	X	X	X	X
C. Maps, Charts, Graphing	X	X	X	X	X	X
V. Problem Solving						
A. Skills and Analysis	X	X	X	X	X	X
B. Application	X	X	X	X	X	X
VI. Geometry						
A. Points, Lines, Angles, Planes			X	X	X	X
B. Closed Plane Figures		X	X	X	X	X
C. Closed Space Figures		X	X	X	X	X
D. Measurement		X	X	X	X	X
E. Construction					X	X
F. Congruence					X	X
G. Similarity					X	X

Number and Numeral

Counting

Counting in its simplest form is mere rote counting, but counting may be so developed as to include grouping. There are five stages in the complete process of counting: (1) rote counting, (2) enumeration or rational counting, (3) reproduction, (4) comparison, and (5) grouping.

a. Rote Counting requires only the saying of the number names in correct sequences. Example: 1,2,3,4,5,6,7,8,9,10, etc. It does not involve the understanding of what the number stands for. The child is merely verbalizing.

b. Enumeration or rational counting tells how many objects there are in a set. This requires the child to identify the number with a specific set of objects. As the child touches each object he associates the correct number name with it. Example: A set of four blocks are on the table. The child is asked to tell how many blocks there are in the set. As he counts 1,2,3,4, - he touches one block at a time and tells you that there are a total of 4 blocks, not the 4th block, in this case. If the child has difficulty with enumeration lead him to see one-to-one relationships. This involves matching, not counting. Use a set of objects and have the child match another set, one at a time to the first set. For example, match a group of children with a set of chairs, to show in this case that for every child there is a chair.

c. Comparison is the noting of differences in regard to quantity, size, weight, distance, and time. Numbers are used to denote this comparison. They themselves do not possess "quantity", "weight," "size," "distance," or "time." Exercises include: Which numbers denote the larger quantity than 6: 1,3,8,9,5, 4,7,? Which numbers denote the smaller quantity than 7: 2,5, 6,9,3, or 4? Comparison should be of two types: exact, where the child gains the understanding that there are two more in this set than in the other, or this set has three less than the other set; and crude, where the child develops the use of estimation in comparing the size, weight, distance, or time, of different sets of objects.

d. Reproduction is the selection of a certain number of objects from a larger set of objects. The child must understand the one-to-one relationship between numbers and objects to obtain the correct answer. Many experiences should be given the child in developing this ability. Such experiences might be: Show a set of five blocks. Give me a set of nine sticks. Take 5 blocks from that set. Put a set of 7 clothespins on the table.

e. Grouping is the association of a number with a set of objects. Many varied experiences should be provided for the children to recognize numbers in different patterns. It is very important that no one pattern become associated with any one number. Here the child is developing the ability to separate a larger set of objects into smaller sets, and to take smaller sets of objects and combine them into one large set. Activities with materials in which the child takes a recognized set of objects, for example, 5 blocks and discovers that a set of 3 blocks and a set of 2 blocks when combined form a set of 5 blocks; a set of 2 blocks and a set of 3 blocks when combined form a set of 5 blocks; a set of 4 blocks and a set of 1 block when combined form a set of 5 blocks; and a set of 1 block and a set of 4 blocks when combined form a set of 5 blocks; such activities contribute to the overall readiness for arithmetic instruction and develop a meaningful understanding of numbers.

The various counting skills and understandings to be developed include the following. The child should be able to:

Grade I

(1) Count by rote to 100.
(2) Count by rote to 100 beginning with any numerical less than 100.
(3) Count by 2's, 5's, 10's, to 100.
(4) Count by odd and even numbers through 100.
(5) Count ordinal numbers through tenths.
(6) Count rationally beginning with any number less than 100.
(7) Use tally marks to record the number of objects in a set.
(8) Determine the actual number of objects in a set of 100 or less by counting. (rational counting)
(9) Recognize at a glance sets of 2,3,4, or 5 objects arranged in varying patterns and be able to tell the total number of objects in the set.
(10) Recognize at a glance 6,7,8,9, or 10 objects arranged in varying patterns and be able to quickly give the total of objects in the group.

Grade II

(1) Count by ones, two, fives, tens, and one hundreds to 1,000.
(2) Count rationally up to 1,000 by beginning with any number less than 1,000.
(3) Determine at a glance the number of objects in randomly arranged groups (number of objects not exceeding 10) and explain how the cardinal number was determined.

(4) Count the number of objects in a group and determine without regrouping whether that group can be divided into two equal parts.
(5) Count by tens beyond 100 by beginning with any number from 1 through 9.
(6) Count by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's, 10's, 100's, 1000's, beginning with any number which may be given.

Grades III Through VI

(1) Maintain previous learnings.

Multiple counting is developed as a means of faster counting. Opportunities of many and varied experiences wherein the skill can be developed are the key~~'~~s to the success of it's presentation. Crayons, chalk, blocks, clothespins, books, sticks, and later on the abacus, the hundred's board, and the number line are just a few of the many materials that can be used to develop meaning and understanding of the skill. As a first step in multiple counting the children must be able to count meaningfully, know that a number measures a set, understand the serial order of numbers, and be able to count by 10's. The understandings and skills that should be developed are included in the preceding overall scope and sequence of counting.

Number and Numeral

Reading, Writing, and Meaning of Numbers and Numerals

These skills and understandings should be introduced and taught early in the 1st grade. The prerequisite for gaining the meaning of numbers is the ability to count before attempting to work with the concepts of an individual number. There are many ways in which a child can be guided to discover the various understandings.

The difference between "number" and "numeral" must be understood. Develop the concept that number is an idea we get by looking at the number of objects in a set. Numeral is the symbol. It is something we can read, write, see, and erase. We use this symbol to represent a number idea.

Numeral	Number Idea
3	0 0 0

Develop the concept that each number has many names - different names for the same number idea.

Number Idea	Names
0 0 0	3; 2+1; 1+2; 0+3; 3+0; 5-2; 4-1; etc.

Introduce and develop the use of expanded numerals.

$9 = 5 + 4$; $23 = 20 + 3$; $23 = 2$ tens $+ 3$ ones

$432 = 400 + 30 + 2$; $526 = 5$ hundreds $+ 2$ tens $+ 6$ ones

$$\begin{array}{rrrrrrr} & 252 & = & 200 & + 50 & + 2 \\ + & 227 & = & 200 & + 20 & + 7 \\ \hline & 479 & = & 400 & + 70 & + 9 \end{array}$$

ETC.

The reading and writing of numbers is developed along with the development of meaning through pictures and objects. It is important throughout the entire developmental period that the children work with a variety of concrete and semi-concrete materials, along with the abstract symbol.

The various reading and writing skills and understanding to be developed include the following. The child should be able to:

Grade I

(1) Read and write numerals by ones, twos, fives, and tens to 100.
(2) Read and write number words through 10.
(3) Read ordinal words through tenth.

Number and Numeral

Reading, Writing, and Meaning of Numbers and Numerals

Grade I - continued

(4) Identify odd and even numerals.
(5) Read and write number names through 100.
(6) Read ordinal names through 20th.

Grade II

(1) Write thousands, hundreds, tens, and ones in columns.
(2) Read number words through thousands.
(3) Relate "teens" and decade number words.
(4) Use symbols and words for one-half, one-third, two-thirds, three-fourths, one-fourth, and one-fifth.
(5) Use ordinals correctly as needed in situations.
(6) Write numerals through 1000.
(7) Read numerals through 1000 without using the word "and".

Grade III

(1) Write thousands, hundreds, tens, and ones in columns.
(2) Read and write fractions.
(3) Read number words through thousands.

Grade IV

(1) Read and write numerals and number words through millions.
(2) Read and write fractions, decimals, percents.

Grade V

(1) Read and write numerals and number words through billions.
(2) Read and write decimals numerals through hundredths.

Grade VI

(1) Read and write decimal numerals through thousandths.
(2) Read and write numerals for positive and negative integers.

Number and Numeral

Positional Value - (place value)

The teaching of 10 as the first two-place numeral is important as it lays the foundation for understanding that our number system is based on ten. If a child can associate the idea of units with the numbers 0 through 9, as 4 units, 7 units, 9 units, he will be able to understand the meaning of zero in the numeral 10. By combining ten sticks into a set of 10 the child discovers that he has one ten and no units, hence he writes this in abstract form as 10. In this case the zero denotes absence of "unit" quantity.

As he adds a stick at a time to his set of ten, he discovers that he has 10 + 1 = 11, 10 + 2 = 12, 10 + 3 = 13, etc. This is the beginning of understanding positional value. He could write the 12 sticks as 12 units, but with our number system with its base of ten, he writes abstractly the numeral 12, showing that he has one set of ten sticks and a set of two sticks more. Other materials, such as pictures or dots and dashes on cards or on the blackboard, could be used to develop the idea of grouping 10's and 1's.

Encourage the children to record their findings in picture form: 16 might call to mind ////////// //////, ///// ///// ///// /. Further understanding would lead to the following exercises:

16 is ____ ten and ____ units.   ____ is 1 ten and 6 units.

With this basic foundation of 10 as the first two-place numeral and the understanding of the "teen numbers", the successive decades can be developed. The children will continue to use the same materials that were used in objectifying the numbers, and in addition will use the abacus, the hundreds chart, and the number line.

The specific place value skills and understandings that should be developed include the following. The child should be able to:

Grade I

(1) Change any number of objects (not exceeding a hundred) into as many sets of ten as possible.

(2) Write the appropriate numeral (10, 20, 30, etc.) when it is evident that there is 1 set of ten, 2 sets of ten, and so on up to 9 sets of ten.

(3) Attach the written numeral to any number of objects from 1 through 100 when objects are arranged in groups of tens and units:

....
.... depicts 24 (without counting by ones)

(4) Count by tens and finish by counting by units the number of elements in a set one hundred or less (if number is not divisable by ten)

Number and Numeral

Positional Value - (place value)

Grade I - continued

(5) Reproduce with counters or other objects in units of tens and ones, any numeral from 1 to 100.

(6) Use tally marks or equivalent in a specially designated place to record the number of subsets of ten in a set and another group of tally marks in a different place to record the remaining subset of elements.

(7) Determine by inspection of a two-digit number the greatest number of tens it contains and how many units as a remainder.

(8) Use zero as a place holder.

Grade II

(1) Write the appropriate numeral (110, 120, etc.) when it is apparent that there are 11 groups of 10, 12 groups of 10, and so on.

(2) Write the appropriate numeral when it is apparent that there are 2-5 hundreds. Use of place value chart and abacus as means of showing.

(3) Represent on the abacus, place value chart, etc. any number of elements up to 500 by the least number of beads or tally marks.

(4) Reproduce with counters or other objects in units of hundreds, tens, and units, any numeral through 500.

(5) Determine by inspection the number of hundreds, tens, or units in a given number up to 500 when the number is given in numerals or is represented on a abacus.

152 contains:	1 hundred	52 left over
	15 tens	2 left over
	152 units	

Grade III

(1) Show correctly on a money place value chart any amount of pennies through 500.

(2) Show correctly on a place value chart the number of pennies on a specific amount of money given in dollars and cents.

(3) Write the appropriate numeral when it is apparent that there are 51-500 sets of ten; that there are 6-50 sets of hundreds; that there are 1-5 sets of thousands.

Number and Numeral

Positional Value (place value)

Grade III - continued

(4) Correctly use the zero symbol when writing the numerals corresponding to the number represented by the presence or absence of beads in the columns on an abacus.

(5) Write the appropriate numeral when it is apparent that there are 0-5 thousands, 0-9 hundreds, 0-9 tens, and 0-9 units.

(6) Reproduce with counters or other objects in sets of hundreds, tens, and units any numeral through 5000.

(7) Represent on an abacus, place value chart, etc. any number of objects or any numeral up to 5000 by the least number of beads or tally marks.

(8) Determine by inspection the number of units, tens, hundreds, or thousands in a given number up to 5000 when the number is given in numerals or represented on an abacus.

Grade IV

(1) Determine by inspection of a three or four digit numeral, the equivalence of values which the digits represent:

Example: 728: 7 hundreds and 2 tens and 8 units or 72 tens and 8 units. 7936 equals 793 tens and 6 units or 79 hundreds and 36 units or 7 thousands and 936 units.

(2) Same as above but with a five digit numeral.

(3) Read, write, and recognize zero as a symbol representing at least two meanings:
the absence of quantity--"not any."
to hold a position open.

Grade V

(1) Determine by inspection a six digit numeral; how many tens there are in the ten's place, hundreds in the hundred's place, thousands in the thousand's place, etc.

Grade VI

(1) Represent five or more digit numerals on an open end abacus, value chart, etc., in several ways.

(2) Reconstruct a five to ten digit numeral by any method to illustrate the principle used when regrouping.

Sets and Sentences

Sets

The specific skills and understandings regarding sets that should be developed include the following. The child should be able to:

Grade I

a. Develop the idea that a set is a collection or group of objects.

b. Know that objects in a set are called its members or elements.

c. Know the number assigned to a set is determined by counting the members or elements of the set.

d. Know that the empty set has no members. Zero is the number assigned to the empty set.

e. Do the following activities:

(1) Find and name sets
(2) Compare sets
(3) Complete sets
(4) Match elements of sets in one-to-one correspondence
(5) Combine and separate sets
(6) Find and recognize sets

Grade II

a. Build on concepts introduced in the first grade

b. Continue to:

(1) Match sets in one-to-one correspondence.
(2) Compare sets
(3) Complete sets
(4) Find and name sets
(5) Combine and separate sets
(6) Find and recognize sets

Grade III

a. Build on concepts introduced in the previous grades

b. Continue to:

(1) Explore and recognize sets
(2) Compare sets
(3) Complete sets
(4) Combine and separate sets
(5) Use set notation or braces to enclose sets

Sets and Sentences

Sets

Grade IV

a. Review set theory

(1) Define sets and subsets

(a) Set - a well defined group or collection
(b) Subset - part of a set
Set A-the set of even numbers, i.e.(2,4,6,8,....)
Set B-subset of set A, i.e. the set of even numbers to six -(2,4,6)

(2) Define elements of sets - The elements of a set are its members, the objects or numbers that belong to it. (2 and 4 are elements of "Set B")

(3) Define equal sets - sets that are identical in size, shape, color, and number or elements.

(4) Define one-to-one correspondence - exists when sets have the same number of members. Sets having one-to-one correspondence are equivalent.

(5) Use braces ($\{\ \}$) to enclose sets

Grade V

a. Review set theory (as developed in previous grade)
b. Understand finite and infinite sets

Grade VI

a. Review set theory (as previously developed)
b. Understand disjoint sets

2. Sentences

Grade I

a. Use open sentences with one and two placeholders

$2 + 2 = \bigcirc$ $\triangle + \square + 3 = 8$

b. Use frames to indicate placeholders

(1) Know if two or more frames are the same in a problem, the like frames represent the same numeral.

$\triangle + \triangle = 8$ $\square + \square = 6$

(2) Know if frames are different in problem, they represent different numerals.

$\triangle + \square = 8$

c. Use equations and inequalities

(1) Know that equations exist when expression on both sides of the equal sign are names for the same number.

$6 = 4 + 2$ $(2 + 1) + 1 = \square$ $2 + 1 = 6 - \square$

Sets and Sentences

Sentences

Grade I - continued

c. Use equations and inequalities

(2) Uses symbols for inequality - greater than and less than.

$10 > 9 \quad 4 < 8 \quad 5 + 3 > 6 \quad 2 + 2 < 8 - 3$

Grade II

a. Use open sentences with one and two placeholders.

$8 - \square = 5 \qquad 9 + \bigcirc + \triangle = 12$

b. Use frames to indicate placeholders.

(1) Know if the frames are the same in a problem; the numerals are the same.

$\triangle_{4} + \triangle_{4} = 8$

(2) Know if frames are different in the problem; they represent different numerals.

$\boxed{5} + \triangle_{2} = 7$

c. Use equations and inequalities.

(1) Build on equation concepts introduced in previous grade.

(2) Build on inequality concepts introduced in previous grade.

(3) Understand the symbol of $\neq$ as meaning not equal to.

Grade III

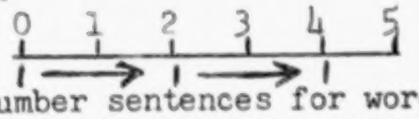
a. Use open sentences with one or two placeholders.

(1) Build on concepts introduced in previous grades.

b. Use frames to indicate placeholders.

(1) Build on concepts introduced in previous grades.

c. Use the number line to prove addition and subtraction facts.



$2 + 2 = 4$

d. Use number sentences for word problems, using addition, subtraction, multiplication, and division.

$34 + 38 = n \qquad n = 72$
$36 + n = 49 \qquad n = 13$
$9 + 3 = n \qquad n = 12$
$15 \div 3 = n \qquad n = 5$
$9 \times 3 = n \qquad n = 27$

Sets and Sentences

Sentences

Grade IV

a. Use frames and letters as placeholders.

  (1) Know that frames or letters are used to hold the place of the missing numeral.

  $\square \times \square = 36 \longrightarrow n \times n = 36$

  $\square \times 10 = 90 \longrightarrow y \times 10 = 90$

  (2) Know that open sentences contain placeholders that cannot yet be judged to be true or false.

b. Determine true and false mathematical statements.

$\triangle \times 52 = 624$ Is the following true or false? $\triangle\hspace{-0.9em}{\scriptstyle 12} \times 52 = 624$

c. Use equations and inequalities.

  (1) Know that equations are number sentences which express an equal relationship. Number represented on one side of equal sign must be the same as that represented on the otherside of the equal sign.

  (2) Know that inequalities are number sentences that do not express an equal relationship.

d. Use the following symbols in sentences to determine relationships of numbers.

$=$	$\neq$	$<$	$>$	$\ngtr$	$\nless$
equal to	not equal to	less than	greater than	not greater than	not less than

Grade V

a. Build on previous learned concepts.

b. Use the symbol $\leqq$ as meaning "is less than or equal to."

c. Use the symbol $\geqq$ as meaning "is greater than or equal to."

d. Use letters, frames, or any other suitable symbol which stands for the missing numeral as placeholders.

Grade VI

a. Review previous learnings.

b. Derive formulas and generalizations from ideas of geometric formulas.

addend + addend = sum   factor x factor = product

$x + y = z$ $A \times B = C$

Whole Numbers

Properties & Relationships

The specific skills and understandings regarding the various properties and relationships with whole numbers include the following. The child should be able to:

Grade I

a. Understand the Commutative Property of Addition. (the order of the addends does not change the sum)

$2 + 4 = 6 \qquad 4 + 2 = 6$

b. Understand the Associative Property of Addition. (regardless of which two numerals are added first, the sum remains the same.

$2 + 1 + 3 + 4 = 10, \quad 1 + 4 + 2 + 3 = 10, \quad 4 + 3 + 2 + 1 = 10$

c. Understand the inverse operation with addition and subtraction.

$3 + 1 = 4 \qquad 4 - 1 = 3$

$1 + 3 = 4 \qquad 4 - 3 = 1$

d. Understand the relationship between facts (patterns)

(1) Show that when 1 is added to a number, the sum is the next counting number.

$$\begin{array}{r}1\\+1\\\hline 2\end{array}\quad\begin{array}{r}2\\+1\\\hline 3\end{array}\quad\begin{array}{r}3\\+1\\\hline 4\end{array}\quad\begin{array}{r}4\\+1\\\hline 5\end{array}\quad\begin{array}{r}5\\+1\\\hline 6\end{array}$$

(2) Show that when 1 is subtracted from a number, the remainder is the next lowest number.

$$\begin{array}{r}2\\-1\\\hline 1\end{array}\quad\begin{array}{r}3\\-1\\\hline 2\end{array}\quad\begin{array}{r}4\\-1\\\hline 3\end{array}\quad\begin{array}{r}5\\-1\\\hline 4\end{array}$$

(3) Show that when 0 is added or subtracted from a number, the number remains the same.

$$\begin{array}{r}2\\+0\\\hline 2\end{array}\quad\begin{array}{r}3\\+0\\\hline 3\end{array}\quad\begin{array}{r}3\\-0\\\hline 3\end{array}\quad\begin{array}{r}2\\-0\\\hline 2\end{array}$$

e. Explain that zero is the identity element for addition. The sum of zero and any other number is the same number,

Grade II

a. Review previous learnings.

b. Understand the Commutative Property of multiplication.

c. Understand the relationship of multiplication to addition (multiplication can be shown by repeated additions.

$2 + 2 + 2 + 2 = 8$ 4 two's are 8

d. Show that the identity element of multiplication is 1 (One times any number is the same number)

$1 \times 3 = 3, \quad 1 \times 4 = 4, \quad 1 \times 5 = 5$

Whole Numbers

Properties & Relationships

Grade III

a. Review previous learnings.

b. Understand the distributive property of multiplication over addition. (The product of a number and the sum of two numbers is the sum of the products of the first number and each of the others.)

$$\begin{aligned} (4 \times 8) = 4 \times (5 + 3) &= (4 \times 5) + (4 \times 3) \\ &= \quad 20 \quad + \quad 12 \\ &= \quad 32 \end{aligned}$$

c. Understand the relationship of division to subtraction. (Division can be shown as repeated subtractions)

$$\begin{array}{rl} 12 & \\ \underline{-3} & (1) \\ 9 & \\ \underline{-3} & (2) \\ 6 & \\ \underline{-3} & (3) \\ 3 & \\ \underline{-3} & (4) \\ 0 & \end{array} \qquad 12 \div 3 = 4$$

d. Understand the inverse operations of multiplication and division. (Division "undoes" multiplication and multiplication "undoes" division)

$$\begin{array}{ll} 4 \times 3 = 12 & \qquad 12 \div 3 = 4 \\ 3 \times 4 = 12 & \qquad 12 \div 4 = 3 \end{array}$$

Grade IV

a. Review previous learnings.

b. Continue to develop concepts of Commutative Properties of Addition and Multiplication.

c. Continue to develop concepts of the Associative Property of Addition.

d. Understand the Associative Property of Multiplication. (Factors may be grouped in any way without changing the product.)

$(2 \times 3) \times 4 = 24$ or $2 \times (3 \times 4) = 24$

so $(2 \times 3) \times 4 = 2 \times (3 \times 4) = 24$

$2 \times (3 \times 5) = 2 \times 15 = 30$

Whole Numbers

Properties & Relationships

Grade IV - continued

e. Understand the Distributive Property of Multiplication over addition.

$$\begin{aligned} 4 \times 26 &= 4 \times (20 + 6) \\ &= (4 \times 20) + (4 \times 6) \\ &= 80 + 24 \\ &= 104 \end{aligned}$$

$$\begin{aligned} 4 \times 67 &= n \\ 4 \times (60 + 7) &= n \\ (4 \times 60) + (4 \times 7) &= n \\ 240 + 28 &= n \\ 268 &= n \end{aligned}$$

f. Develop meaning and usage of factors.

$1 \times 9 = 9$
$9 \times 1 = 9$ Factors 1, 3, 9,
$3 \times 3 = 9$

Grade V

a. Understand factors and primes.

(1) 3,4; 6,2; 12,1 are all factors of 12 since 3×4, 6×2, $12 \times 1 = 12$.

(2) 3 is a prime number, prime because its only factors are itself and 1 (only $3 \times 1 = 3$)

(3) Use tests for divisibility.

b. Understand and use each of the basic properties:

(1) Use commutative property of addition and multiplication.

(2) Use associative property of addition and multiplication.

(3) Use distributive property of multiplication over addition.

(4) Use inverse operations (addition-subtraction) (multiplication-division)

(5) Use properties of zero and one.

(a) Multiplicative identity: $1 \times A = A$

(b) Additive identity: $0 + B = B$

(c) Zero as a placeholder.

(6) Understand the relationship of division to subtraction.

(7) Understand the relationship of addition to multiplication.

Grade VI

a. Review previous learnings.

b. Express properties in algebraic form.

c. Understand closure under addition and multiplication.

d. Understand the principle of reciprocals in working with fractions.

Whole Numbers

Operations

Addition

The specific addition skills and understandings that should be developed include the following. The child should be able to:

Grade I

(1) Demonstrate with counters or other objects what is meant by "counting together" groups.
(2) Use concrete objects to arrive at answers to the addition combinations with sums through 10.
(3) Symbolize in arithmetic language when a "counting together" action is demonstrated by another person.

$4 + 2 = 6$ or $\begin{array}{r}4\\+2\\\hline 6\end{array}$ is written after counting together action takes place.

(4) "Count together" groups when + is shown.

$2 + 3 = 5$ or $\begin{array}{r}2\\+3\\\hline 5\end{array}$

(5) Tell a story problem to illustrate a number combination. Example: given 2 + 3 = 5. Matt tells "I have two cookies in one hand and three cookies in the other. How many cookies do I have altogether?
(6) Give the answer to a combination such as 2 + 5 immediately after obtaining the sum of 5 + 2 by the use of concrete or semi-concrete materials.
(7) Demonstrate mastery of addition combinations with sums through 10 by responding automatically and using correctly in problem situations.
(8) Use **=** as meaning "are" or "equal". Example: 2 and 2 are 4; 2 and 2 equal 4; read facts such as 2 + 2 = 4 , as 2 and 2 equal 4, or 2 and 2 are 4.
(9) Write 2 + 2 = 4 when 2 and 2 equal 4, or 2 and 2 are 4 is read or is heard.
(10) Use concrete objects to arrive at answers of the addition combinations through 18.
(11) Find the sum of a single column of addends not exceeding 3 whose sum does not exceed 10.
(12) Demonstrate mastery of addition combinations with sums through 18 by responding automatically and using correctly in any situation.
(13) Demonstrate the mastery of the doubles with sums through 18. 7 + 7 = 14, 9 + 9 = 18.
(14) Demonstrate the mastery of ten plus any number.

10 + 3 = 13　　30 + 6 = 36
10 + 7 = 17　　50 + 3 = 53
10 + 9 = 19　　90 + 7 = 97

Whole Numbers

Operations

Addition - continued

(15) Transform more difficult combinations whose sums are not more than 18 into simpler sums that are already known.

Example: $7 + 8 = 10 + 5 = 15$
$7 + 8 = 7 + 7 + 1 = 15$
$7 + 8 = 8 + 8 - 1 = 15$

(16) Show with counters or other material how many objects are in 1 through 10 groups of 2; and how many groups of 3 are in any given number 1 through 9.

(17) Use concrete objects to arrive at answers to those subtraction combinations which correspond to the addition combinations with sums through 18.

(18) Find the number which takes the place of the symbol N, △, □, ○, ▨, etc. which will make statements like the following true.

Example: $n + 3 = 9$
$9 - \triangle = 3$
$6 + 3 = \square$
9 - ▨ = 6

Grade II

(1) Add a two-digit number and one-digit number where the sum of end digits is the same as when the end digits are added alone:

$$\begin{array}{r} 26 \\ \underline{+3} \\ 29 \end{array} \qquad \begin{array}{r} 6 \\ \underline{+3} \\ 9 \end{array} \qquad \text{(adding by endings)}$$

(2) Determine by use of concrete objects sums when the algorism containing three addends are given in both vertical and horizonal form:

Example: $2 + 3 + 1 = n$

$$\begin{array}{r} 2 \\ 3 \\ \underline{+1} \end{array}$$

is given and sum determined by "counting together."

(3) Determine quickly the sum of any one digit number and any two digit number divisible by 10.

Example:
$$\begin{array}{r} 50 \\ \underline{+8} \end{array} \qquad 70 + 6 = 76$$

(4) Determine quickly the sum of any two-digit number less than 20 and any number (through 80) divisible by 10.

Example:
$$\begin{array}{r} 30 \\ \underline{+17} \\ 47 \end{array} \qquad 60 + 15 = 75$$

(5) Find the sum of double column of addends whose sum exceeds 10 (involving only combinations they have mastered)

Example:
$$\begin{array}{r} 43 \\ \underline{+27} \\ 60 \\ \underline{10} \\ 70 \end{array} \qquad \begin{array}{r} 43 \\ \underline{+27} \\ 10 \\ \underline{60} \\ 70 \end{array}$$

Whole Numbers

Operations

Addition - continued

(6) Find the sum of a double column of addends not exceeding three, whose sum does not exceed 10. Example:

$$\begin{array}{r}13\\21\\+\underline{14}\\48\end{array}\qquad\begin{array}{r}13\\21\\+\underline{14}\\40\\\underline{8}\\48\end{array}\qquad 13+21+14=48$$

(7) Add two or three digit addends when the sum of all the tens is greater than 10. Example:

$$\begin{array}{r}40\\+\ \underline{80}\\120\end{array}\qquad\begin{array}{r}34\\91\\+\underline{32}\\157\end{array}$$

(8) Add with zeros in the addends. Example:

$$\begin{array}{r}20\\+\underline{29}\\49\end{array}\qquad\begin{array}{r}3\\0\\+\underline{2}\\5\end{array}\qquad\begin{array}{l}3+0+2=5\\\\29+20=49\end{array}$$

Grade III

(1) Demonstrate mastery of the addition facts whose sums are less than 19 by responding automatically and using correctly in practical situations.

(2) Add by endings in column addition, using not more than five addends. Example:

$$\begin{array}{rl}3&(17)\\5&(14)\\4&(\ 9)\\1&(\ 5)\\\underline{4}&\\17&\end{array}\qquad 3+\overset{(8)}{5}+\overset{(12)}{4}+\overset{(13)}{1}+\overset{(17)}{4}=17$$

(3) Place each numeral of the addends and sum in its correct position (place). Example:
The ones in the one's place, tens in the ten's place, hundreds in the hundred's place.

(4) Add two or three addends (two digits) when the sum of the digits in the ones place is greater than 10. Use of partial sum method. Example:

$$\begin{array}{r}14\\+\underline{27}\\11\\\underline{30}\\41\end{array}\text{ or }\begin{array}{r}14\\+\underline{27}\\30\\\underline{11}\\41\end{array}\qquad\begin{array}{r}34\\+\underline{89}\\13\\\underline{110}\\123\end{array}\text{ or }\begin{array}{r}34\\+\underline{89}\\110\\\underline{13}\\123\end{array}$$

(5) Add with carrying in one's place, changing ones to tens.
Example:

$$\begin{array}{r}14\\27\end{array}\qquad\begin{array}{r}98\\46\\\underline{52}\\196\end{array}$$

Whole Numbers

Operations

Addition - continued

(6) Add a column of numbers with zero as a digit.
Example: $\begin{array}{r}213\\210\\17\\\hline 440\end{array}$

(7) Place cents point and dollar sign correctly when adding.
Example: $\begin{array}{r}\$1.25\\1.19\\\hline \$2.44\end{array}\qquad\begin{array}{r}\$\ .78\\.34\\\hline \$1.12\end{array}\qquad\begin{array}{lr}\$ & .78\\ & .12\\\hline \$ & .90\end{array}$

(8) Find sums of two and three addends when the sum of the digits in the ten's place is greater than 10, and the sums of the digits in the one's place and hundred's place may or may not be greater than 10.
Example:

$$\begin{array}{rcrrcr}183 & & 183 & 682 & & 682\\135 & & 135 & 175 & & 175\\281 & & 281 & 281 & & 281\\\hline 9 & & 400 & 8 & & 900\\190 & \text{or} & 190 & 230 & \text{or} & 230\\400 & & 9 & 900 & & 8\\\hline 599 & & 599 & 1138 & & 1138\end{array}$$

(9) Add with carrying (regrouping) in the ten's place, changing the tens to hundreds with three digit numbers.
Example: $\begin{array}{r}146\\271\\\hline\end{array}\qquad\begin{array}{r}247\\352\\171\\\hline 770\end{array}$

(10) Add one or two digit numbers with not more than five addends by forming decades:
Example:

$$\begin{array}{lrl}10\ (4 + 3 = 7) & 35 & 20 + 6\\ & 68 & 10\ (3 + 7 = 10)\\4 + 4 + 2 = 10 & 44 & \\ & 29 & \quad 9 + 1 = 10\\ & \overline{176} & \end{array}$$

(11) After determining the answer to an addition exercise, respond quickly when an addend is increased or decreased by some multiple of ten.
Example:
Determine that 27 + 7 is 34 after knowing 17 + 7 = 24

(12) Add 2,3,4, or more place numbers with at least four addends.

Grade IV

(1) Add two-place numbers to one place numbers when the sum falls:
(a) within the decade, 22 + 5 = 27
(b) at the decade, 22 + 8 = 30
(c) into the next decade, 22 + 9 = 31

Whole Numbers

Operations

Addition - continued

Grade IV

(2) Use addends which are composed of dollars and cents.
Example:
$106.59
75.24
.64
$182.47
(Stress importance of keeping the decimals in a straight column so uneven columns may be added correctly)

(3) Add two-place, three-place, and four-place numbers in regular and irregular columns with and without regrouping.
Example:
1986
27
357
4
2374

Grade V

(1) Estimate the reasonableness of the answer in an addition problem:
Example:
725 (is about 700)
+352 (is about 400)
(answer is less than 1100, answer = 1077

(2) Add 5 (or more) digit numbers with 5 addends, involving both regular and irregular columns.
Example:
62,784
470
29
8,784
55,576
127,643

Grade VI

(1) Maintain previously learned skills and understandings.

Whole Numbers

Operations

Subtraction

The specific subtraction skills and understandings that should be developed include the following. The child should be able to:

Grade I

(1) Demonstrate with counters or other objects what is meant by separating (counting away) groups.
(2) Solve a story problem involving separating groups when a story is told by someone else.
(3) Separate groups when the (-) sign is shown.
Example: 5 - 2 = 3

$$\begin{array}{r} 5 \\ -\,2 \\ \hline 3 \end{array}$$

(4) Use concrete objects to arrive at answers to the subtraction combinations which correspond to the addition combinations with sums through 10.
(5) Indicate immediately after obtaining the answer to a subtraction combination as 4 - 2 = N, that 2 - 4 = N does not make sense.
(6) Symbolize in arithmetic language the separating of a subset from the total set as demonstrated by another person.
Example: 7 - 4 = 3 or

$$\begin{array}{r} 7 \\ -\underline{4} \\ 3 \end{array}$$

written after counting away action takes place.
(7) Demonstrate mastery of subtraction combinations by responding automatically and using correctly in any situation. (those corresponding to addition combinations through 6)
(8) Solve by picture or manipulative material a story problem involving the comparison concept of subtraction.
Example: I have a brother who is 8 years old. I am six years old. How much older is my brother than I? (How much younger am I than my brother?)
(9) Solve by picture or manipulative materials a story involving the "lack" concept of subtraction.
Example: We have ten children in our group. Here are three books. How many more do we need?

Grade II

(1) Identify and demonstrate the three meanings of subtraction; (a) remainder - 6 birds are on a wire, 4 flew away. How many are left? (b) comparison - 6 birds are on a wire, 4 on another. How many more (or less) birds are on one wire than the other? (c) lack - 4 birds are on a wire. How many more are needed to have 6 birds on the wire?
(2) Demonstrate mastery of subtraction combinations through 10 by responding automatically and using correctly in all situations.

Whole Numbers

Operations

Subtraction

Grade II - continued

(3) Transform the more difficult subtraction combinations (corresponding to the addition combinations whose sums are no more than 18) into simpler combinations already known. Example: $17 - 9 = 10 - 9 + 7 = 8$

$17 - 8 = 10 - 8 + 7 = 9$

$17 - 9 = 18 - 9 - 1 = 8$

(4) Indicate from a list of number pairs separated by a minus sign those pairs in which subtraction is possible and those which are not.
Example: $9 - 8 = n$, possible $7 - 10 = n$, not possible

(5) Subtract two-digit numbers without re-grouping.
Example:

$$\begin{array}{r} 26 \\ -13 \\ \hline 13 \end{array}$$

including zero in the ones column

(6) Solve together with the teacher a simple problem which arises from group situations involving regrouping in subtraction or renaming in addition.
Example: There are 24 children in our room. Only 19 are here today. How many are absent?

(7) Demonstrate mastery of subtraction combinations by responding automatically and using correctly in all situations.

(8) Place each numeral of the minuend, subtrahend, and remainder in its correct place.
Example: ones in the one's place, tens in the ten's place, hundreds in the hundred's place, etc.

Grade III

(1) Subtract by endings:
Example:

$$\begin{array}{r} 18 \\ -6 \\ \hline 12 \end{array} \qquad \begin{array}{r} 46 \\ -2 \\ \hline 44 \end{array}$$

(2) Subtract with zeros in subtrahend and minuend involving no regrouping.
Example:

$$\begin{array}{r} 205 \\ -105 \\ \hline \end{array} \qquad \begin{array}{r} 235 \\ -101 \\ \hline \end{array}$$

(3) Place cents point and dollar sign correctly when subtracting.
Example:

$$\begin{array}{r} \$2.36 \\ 1.22 \\ \hline \$1.14 \end{array}$$

(4) Recognize that ones in subtrahend cannot be subtracted from ones in the minuend if the ones in the subtrahend are greater than ones in the minuend. Decrease the tens by one ten and convert this ten to ones. Combine these ones with those already in the ones place.

Whole Numbers

Operations

Subtraction

Grade III - continued

(5) Recognize that the tens in the subtrahend cannot be subtracted from tens in the minuend if the tens in the subtrahend are greater than the tens in the minuend. Decrease hundreds by one hundred and convert this hundred into ten tens. Combine these tens with those already in the ten's place.

Example:

$$\begin{array}{r} 314 \\ -142 \\ \hline 172 \end{array}$$

(6) Determine the answer to a subtraction exercise and quickly respond when the minuend is increased or decreased by some multiple of ten.

Example: Determine that 26 - 3 = 23 after knowing 16 - 3 = 13.

Grade IV

(1) Subtract four place numbers with regrouping in tens, hundreds, and thousands column.

Example:

$$\begin{array}{rrrrr} 5132 & 4 \text{ thousand,} & 10 \text{ hundred,} & 12 \text{ tens,} & 12 \text{ ones} \\ -\,4875 & 4 \text{ thousand,} & 8 \text{ hundred,} & 7 \text{ tens,} & 5 \text{ ones} \\ \hline & & 2 \text{ hundred,} & 5 \text{ tens} & 7 \text{ ones} \end{array}$$

$$\begin{array}{rrrrr} 5003 & 4 \text{ thousand,} & 9 \text{ hundred,} & 9 \text{ tens,} & 13 \text{ ones} \\ -2437 & 2 \text{ thousand,} & 4 \text{ hundred,} & 3 \text{ tens,} & 7 \text{ ones} \\ \hline & 2 \text{ thousand,} & 5 \text{ hundred,} & 6 \text{ tens,} & 6 \text{ ones} \end{array}$$

(2) Use minuends and subtrahends involving the use of money.

Example:

Pamela has \$7.80, Matt has \$5.90. How much more money does Pamela have than Matt?

$$\begin{array}{rl} \$7.80 & \text{minuend} \\ 5.90 & \text{subtrahend} \\ \hline \$1.90 & \text{Pamela has more than Matt} \end{array}$$

Grade V

(1) Use amounts of money in subtraction up to five-place figures with regrouping in various positions.

Example:

$$\begin{array}{r} \$311.16 \\ -\ \ 4.89 \\ \hline \$306.27 \end{array}$$

Whole Numbers

Operations

Subtraction

Grade VI

(1) Subtract whole numbers of 4 to 6 digits to include number with all types of difficulties; zero in the tens or (and) hundreds place in either (or both) the minuend and subtrahend.

(2) Orally find the difference between two numbers of two or three digits each -- round the minuend to hundreds than add or subtract.

Example: $\begin{array}{r} 468 \\ -284 \\ \hline \end{array}$ round to $\begin{array}{r} 500 \\ -284 \\ \hline 216 \end{array}$ $\begin{array}{r} 216 \\ -\ 32 \\ \hline 184 \end{array}$

(3) Examine a problem in subtraction and identify it as: (a) take away, (b) comparison, or (c) addition

a. Christy had \$15.00, and lost \$4.00.
b. This board is how much longer?
c. Matt has \$6.00 and he needs \$10.00.

c. Relationships between Addition and Subtraction.

(1) Demonstrate by using number chart, etc. that subtraction has a definite relationship to addition. Addition is the opposite or subtraction (vice versa)

Example:

$6 + 2 = 8$ $8 - 2 = 6$

(2) Write the family of 4 relationships which exist between a given set of 3 members.

Example: For the set 3, 4, and 7, the relationship would be: $3 + 4 = 7$, $4 + 3 = 7$, $7 - 3 = 4$, $7 - 4 = 3$

(3) Show that subtraction can be proved by addition.

Example: $\begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array}$ $\begin{array}{r} 5 \\ +3 \\ \hline 8 \end{array}$

(4) Demonstrate by using manipulative material that each basic process can be interpreted as the rearranging or regrouping of a collection of elements and that all basic process are interrelated.

(5) Show the correctness of addition exercises with up to 5 addends by subtraction.

Example: $\begin{array}{r} 9 \\ +5 \\ \hline \end{array}$ $\begin{array}{r} 14 \\ -5 \\ \hline \end{array}$ $\begin{array}{r} 6 \\ 4 \\ 5 \\ \hline 15 \end{array}$ $\begin{array}{r} 15 \\ -6 \\ \hline 9 \\ -4 \\ \hline 5 \end{array}$

Whole Numbers

. Operations

. Relationship between Addition and Subtraction - continued

(6) Demonstrate the correctness of subtraction exercises by:

(a) Subtract remainder from minuend and the difference should be equal to the subtrahend of the original problem.
Example:

$$\begin{array}{r} 54 \\ -\ 21 \\ \hline 33 \end{array} \qquad \begin{array}{r} 54 \\ -33 \\ \hline 21 \end{array}$$

(b) The sum of the subtrahend and the remainder should equal the minuend.
Example:

$$\begin{array}{r} 54 \\ -33 \\ \hline 21 \end{array} \qquad \begin{array}{r} 21 \\ +33 \\ \hline 54 \end{array}$$

Whole Numbers

Operations

Multiplication

The specific multiplication skills and understandings that should be developed include the following. The child should be able to:

Grade II

(1) Understand the meaning of multiplication through the use of the following discovery techniques:
   (a) Set concepts
   (b) Number line
   (c) Array
   (d) Skip counting
   (e) Repeated additions

(2) Discover (not master) facts and reverses through the use of the discovery techniques and commutative property.

2's through 20	Reverses
3's through 18	
4's through 20	two 4's and four 2's
5's through 18	
7's through 14	4 + 4 = 2 + 2 + 2 + 2
8's through 16	
9's through 18	

(3) Understand that the order of numerals does not affect the product. The symbol for multiplication (x) is not introduced at this level.

(4) Discover facts for 5's through 50 and 10's through 100.

Grade III

(1) Understand presentation methods of multiplication.
   (a) Array
   (b) Repeated addition
   (c) Partial products
   (d) Number line
   (e) Expand form

$$\begin{array}{r} 123 \\ \text{x}5 \\ \hline \end{array} \qquad \begin{array}{r} 100 + 20 + 3 \\ \text{x } 5 \\ \hline \end{array}$$

(2) Discover facts through presentation methods.
   (a) 6's through 60
   (b) 7's through 70
   (c) 8's through 80
   (d) 9's through 90

(3) Understand the meaning and use of signs for multiplication.

(x) multiply: 3 x 2 = n, 3 times 2, 3 2's

(4) Multiply numbers up to 3 digits by 1 digit with and without renaming.

$$\begin{array}{rll} 214 & & \\ \text{x}2 & & \\ \hline 8 & (2 \text{ x } 4) & 214 \text{ x } 2 = (200 \text{ x } 2) + (10 \text{ x } 2) + (4 \text{ x } 2) \\ 20 & (2 \text{ x } 10) & \\ 400 & (2 \text{ x } 100) & \\ \hline 428 & & \end{array}$$

Whole Numbers

Operations

Multiplication

Grade III - continued

(5) Further development.
Multiply one, two, and three place multiplicands by a one-place multiplier without regrouping (partial product method)

$$\begin{array}{r} 4\\ \underline{\text{x}2}\\ 8 \end{array}\qquad \begin{array}{r} 14\\ \underline{\text{x}2}\\ 20\\ \underline{8}\\ 28 \end{array}\qquad \begin{array}{r} 14\\ \underline{\text{x}2}\\ 8\\ \underline{20}\\ 28 \end{array}\qquad \begin{array}{r} 214\\ \underline{\text{x}2}\\ 400\\ 20\\ \underline{8}\\ 428 \end{array}\qquad \begin{array}{r} 214\\ \underline{\text{x}2}\\ 8\\ 20\\ \underline{400}\\ 428 \end{array}$$

(6) Further development.
Multiply two and three place multiplicand by a one-place multiplier with carrying (regrouping) tens to tens.

$$\begin{array}{r} 126\\ \underline{\text{x}2}\\ 252 \end{array}\qquad \begin{array}{r} 126\\ \underline{\text{x}2}\\ 12\\ 40\\ \underline{200}\\ 252 \end{array}\qquad \begin{array}{r} 126\\ \underline{\text{x}2}\\ 200\\ 40\\ \underline{12}\\ 252 \end{array}$$

(7) Further development.
Multiply two and three place multiplicands by a one-place multiplier with regrouping hundreds to hundreds.

$$\begin{array}{r} 173\\ \underline{\text{x}3}\\ 300\\ 210\\ \underline{9}\\ 519 \end{array}\qquad \begin{array}{r} 173\\ \underline{\text{x}3}\\ 9\\ 300\\ \underline{210}\\ 519 \end{array}$$

(8) Multiply a three place multiplicand with a cent point by a one place multiplier.

$$\begin{array}{r} \$1.12\\ \underline{\text{x}\ \ 2}\\ \$2.24 \end{array}$$

Grade IV

(1) Rediscover and learn the facts through the use of arrays, repeated addition, number line, etc.
(2) Relate facts - multiplication and division
3 x 4 = 12   12 ÷ 4 = n
factor x factor=product   product ÷ factor = missing factor
(3) Multiply a 4 digit number by a 1 digit number.
(4) Multiply 2 digit number by a 2 digit number.
(5) Multiply by 10's.
(6) Understand multiplication algorithm.

a. By expanded notation
b. By partial products
c. By standard algorithm

Whole Numbers

Operations

Multiplication

Grade IV - continued

(6) continued

$$\begin{array}{r} 32 \times 2 = 30 + 2 \\ \times\ 2 \\ \hline \underbrace{60}_{\text{partial product}} + \underbrace{4}_{\text{partial product}} = \underbrace{64}_{\text{product}} \end{array}$$

$$\begin{aligned} 2 \times (30 + 2) &= (2 \times 30) + (2 \times 2) \\ &= 60 + 4 \\ &= 64 \end{aligned}$$

$$\begin{array}{r} 32 \\ \underline{\times 2} \\ 64 \end{array} \quad \text{standard algorithm}$$

Grade V

(1) Review and maintain patterns, facts, and arrays.
(2) Review skills for finding missing factors in multiplication.
(3) Review multiplication by 10's, 100's, 1000's.
(4) Change the order of multiplication of factors thus making the process easier.
(5) Estimate the number of places in the answer in multiplication by rounding off the numbers.
(6) Multiply a two-digit multiplicand by a two-digit multiplier with zero in both.
(7) Multiply a three or more digit multiplicand by a two digit multiplier.
Example: Matt made 34 trips to the paper stand to pick-up his newspapers. If he traveled 786 yards each trip, how many yards did he travel?

$$\begin{array}{rl} 786 & \text{yards per trip} \\ \underline{34} & \text{trips} \\ 3144 & \\ \underline{2358} & \\ 26724 & \text{yards traveled} \end{array}$$

(8) Multiply a 5 digit multiplicand by a one-digit multiplier.
(9) Multiply a three digit multiplicand by a three-digit multiplier.
(10) Multiply mentally a two-digit multiplicand by a one-digit multiplier.

Whole Numbers

Operations

Multiplication

Grade VI

(1) Review and maintain patterns, facts, and arrays.

(2) Multiply by three digit multipliers using four place multiplicand, and to include those processed with zero difficulties.

(3) Determine by use of an abacus and later by inspection, the product of a seven digit number by 10; six digit number by 10 or 100; a five digit number by 10, 100, or 1000; a four digit number by 10, 100, 1000, or 10,000, etc.

(4) Multiply by ten or power of ten, orally, by mentally adding zeros or placing a decimal point.

(5) Multiply a two-place number by a two-place number mentally.

Example:

Matt bought 12 tickets to a hobby show. Each ticket cost 24¢. How much did he spend for tickets?

Procedure:

A. You know that 25¢ is 1¢ more than 24¢. Four tickets would cost about $1.00, so tickets would cost 3 x $1.00 or about $3.00. But 12¢ too much has been added so subtract 12¢ from $3.00. That is $2.88.

Procedure:

B. You know that 12 is one ten and two ones. So first multiply 24¢ by 2. That's 48¢. Then multiply 24¢ by 10. That is $2.40. Then add $2.40 and 48¢. That is $2.88.

Procedure:

C. You know that 24¢ is about 25¢. Four tickets would cost $1.00 so tickets would cost 3 x $1.00 or about $3.00. Now find the exact amount by using paper and pencil.

Whole Numbers

Operations

Division

The specific division skills and understandings that should be developed include the following. The child should be able to:

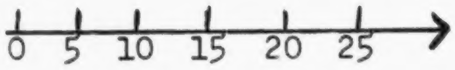
Grade II

(1) Through the use of sets and arrays and as an extension of multiplication, discover how many 2's in 8, etc.

Grade III

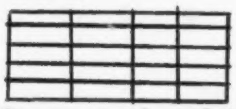
(1) Present methods of division.

a. Number Line



$20 \div 5 = 4$

b. Arrays



$20 \div 4 = 5$
5 rows of 4 in 20

c. Repeated subtraction.

$9 \div 3 = 3$

3 threes in 9

$$\begin{array}{l} 9 - 3 \\ \quad 6 - 3 \\ \qquad 3 - 3 \\ \qquad\quad 0 \end{array}$$

d. Long division algorithm and subtractive algorithm.

$$\begin{array}{r|l} 2\overline{)16} & \\ -8 & (4) \\ \hline 8 & \\ -8 & (4) \\ \hline 0 & \end{array} \qquad \begin{array}{r} 8 \\ 2\overline{)16} \\ 16 \\ \hline 0 \end{array}$$

(2) Understand meaning and use of the division symbol (÷).

(3) Use 1 digit divisor with 3 digit dividend and renaming.

$$4\overline{)156} = 4\overline{)\overset{30\ +\ 9}{120 + 36}} = 39$$

$$4\overline{)\overset{20\ +\ 10\ +\ 9}{80 + 40 + 36}} = 39$$

(4) Understand the use of remainders in division.

a. Determine how many equal sets there are in a number and how many are left over.

$17 \div 2 = 8$ with 1 remainder

b. Check (inverse operation)

$(2 \times 8) + 1 = 17$

(5) Divide by 10.

Whole Numbers

Operations

Division

Grade IV

(1) Review previous learnings.

(2) Use division algorithm.

a. Subtractive algorithm

$$\begin{array}{rll}
25\,\overline{)\,375} & & \\
\underline{-100} & (4) & (4 \times 25 = 100) \\
275 & & \\
\underline{-100} & (4) & (4 \times 25 = 100) \\
175 & & \\
\underline{-175} & \underline{(7)} & (7 \times 25 = 175) \\
 & 15 &
\end{array}$$

b. Transitional (pyramid quotient)

$$\begin{array}{r}
\left.\begin{array}{r} 5 \\ 10 \end{array}\right\} 15 \\
25\,\overline{)\,375} \\
\underline{-250} \\
125 \\
\underline{-125}
\end{array}$$

c. Standard Algorithm

$$\begin{array}{r}
15 \\
25\,\overline{)\,375} \\
\underline{-250} \\
125 \\
\underline{-125}
\end{array}$$

(3) Use remainders in division.

(4) Use zero in dividend and quotient.

(5) Find averages.

(6) Check division by multiplication (inverse operation)

(7) Use the conventional vocabulary for division,

$$\begin{array}{r}
\text{Quotient} \\
\text{Divisor}\,\overline{)\,\text{Dividend}}
\end{array}$$

but make sure to include the terms.

$$\begin{array}{r}
\text{Factor} \\
\text{Factor}\,\overline{)\,\text{Product}}
\end{array}$$

(The basic rule for both multiplication and division is factor x factor = product)

Whole Numbers

Operations

Division

Grade IV - continued

(8) Use mathematical sentences in division process.

(9) Show that problems with remainders are written thus:

$(3 \times n) + r = 26$ also $26 = (3 \times n) + r$

$(3 \times 8) + 2 = 26$ $26 = (3 \times 8) + 2$

(10) Apply division to 2 types of problems:

a. Measurement

24 in. ÷ 3 in.	How many 3 inch strips can be cut from 24 inches of ribbon? (Answer an abstract number)

b. Partition

24 in. ÷ 3	Divide 24 inches of ribbon equally among three people. (Answer a denominate number)

(11) Verify division facts by subtraction.

(12) Divide one, two, or three figure dividend by one figure divisors - even and uneven division.

(13) Express and label numbers "left over" as remainders.

(14) Create a verbal situation that might apply to a given algorism.

(15) Divide a four digit number by 1 and 2 digit number.

Whole Numbers

Operations

Division

Grade V

(1) Divide four and five digit numbers by one and two digit numbers.

(2) Use division algorithm: (Many ways of arriving at an answer should be presented. No one way should be made mandatory during the learning stages. The ultimate goal is to guide toward the use of the refined method)

a. Subtractive

```
23 / 3335
     2300        100 x 23
     1035
      920         40 x 23
      115
      115          5 x 23
                 145
```

b. Transitional

```
         5 }
        40 }        145
       100 }
23 / 3335
     2300
     1035
      920
      115
      115
```

c. Refined method - short division.

```
        145
23 / 3335
     23
     103
      92
      115
      115
```

(NOTE: Use short division only after a complete understanding of the division algorithm)

(3) Estimate quotients.

(4) Express remainders as fractions when appropriate.

```
         4 5/12
12 / 53
     48
      5
```

Whole Numbers

Operations

Division

Grade VI

(1) Understand meaning of division.

(2) Review previous learnings.

(3) Divide four and five digit numbers by two and three digit numbers.

(4) Estimate quotient figures.

(5) Express remainders as fractions, decimal numerals.

$$\begin{array}{r} 4\,\frac{5}{12} \\ 12\,\overline{)\,53} \\ \underline{48} \\ 5 \end{array} \qquad\qquad \begin{array}{r} 4.41 \\ 12\,\overline{)\,53.00} \\ \underline{48} \\ 5\;0 \\ \underline{4\;8} \\ 20 \\ \underline{12} \\ 8 \end{array}$$

(6) Use short division form.

(7) Check division by its inverse operation.

Rational Numbers (fractions, decimals, percentage)

Properties & Relationships

The specific skills and understandings regarding the various properties and relationships with rational numbers include the following. The child should be able to:

Grade I

a. Develop the concept that there are numbers other than whole numbers.

b. Develop an understanding of one-half and one-third of a whole and of a group.

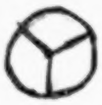
Grade II

a. Review previous learnings.

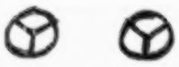
Grade III

a. Understand use of positive rational numbers and zero.

b. Understand unit fractions as applied to whole and to a group.

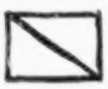


$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$

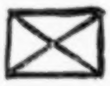


There are 6 thirds in 2.

c. Understand the naming of non-unit fractions with reference to a whole and to a group.



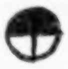
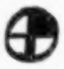
$\frac{2}{2}$



$\frac{4}{4}$

2 ($\frac{1}{4}$ units) are the same as 1 ($\frac{1}{2}$ unit)

Grade IV

a. Develop fractions as part of a whole  $\frac{1}{2}$  $\frac{1}{4}$

b. Develop fractions as part of a set {○○○○} $\frac{1}{2}$ of the set is ○○

c. Develop fractions as naming a ratio {○○△△△} $\frac{2}{3}$ 2 circles to 3 triangles

d. Develop different names for 1 (whole).

$1 = \frac{2}{2}, \frac{4}{4}, \frac{6}{6}, \frac{25}{25}, \frac{100}{100},$

e. Understand fractions and equivalent parts.

$\frac{1}{2}$ equivalent to $\frac{2}{4}$

⊖ ⊕

Rational Numbers (fractions, decimals, percentage)

Properties & Relationships

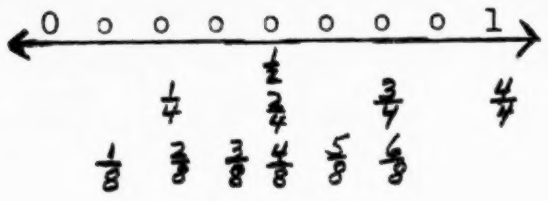
Grade IV - continued

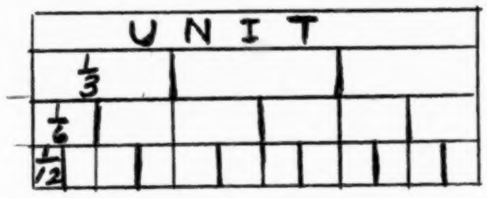
f. Discover relationships between number and size of parts compared to whole.

g. Change fractions to simplest form.

h. Compare fractions.

Use number line and relationship charts to determine fractions = to, < than, > than.





i. Develop commutative property regarding rational numbers.

j. Develop the multiplicative identity property.

k. Understand use of tenths, hundredths, (decimal numerals and percentage)

l. Develop relation of fractions and percents, decimal numerals and percents.

% (percent) means per hundred

50% means $\frac{50}{100}$ or $\frac{1}{2}$

5% means $\frac{5}{100}$

m. Know equivalent forms for fractions and decimals.

Grade V

a. Understand the concept of an infinite set.

b. Differentiate between common denominator; lowest common denominator, least common multiple, common divisor.

c. Simplify fractions and change fractions to mixed numerals.

d. Change terms of fractions.

e. Determine if each of the whole number properties and relationships are true for rational numbers.

Rational Numbers (fractions, decimals, percentage)

Properties & Relationship

Grade VI

a. Understand concept of an infinite (never ending) set.
For every fraction there is always one smaller or larger.

b. Use common denominator, least common denominator, or least common multiple.

(1) 12 is the least common multiple (least common denominator) of 3 and 4 because 12 is the smallest number that can be divided by both 3 and 4 with no remainder.

(2) Common multiples are those numbers divisible by the given numbers. The common multiples of 3 and 4 are 12, 24, 36, ...

c. Simplify and compare fractions.

d. Understand and use ration and proportion.

(1) Ratio is a quotient of two numbers and is a means of comparing them.

(2) Fractions are expressions of ratios.

$$\frac{1}{2} = \frac{2}{4}$$

Example: If 2 pencils cost 5¢, how many pencils can you get for 10¢?

$$\frac{2}{5} = \frac{n}{10} \quad , n = 4$$

e. Define rational number.

f. Determine if each of the whole number properties are true for rational numerals also.

Rational Numbers (fractions, decimals, percentage)

Operations

Generalizations

The specific skills and understandings regarding the various generalizations with rational numbers include the following. The child should be able to:

Grade I

(1) Develop concept that there are numbers other than whole numbers.

(2) Develop understanding of one-half, and one-third of a whole and of a group.

Grade II

(1) Develop understanding of one-half, one-third, one-fourth, one-fifth, two-thirds, and three-fourths, as applied to wholes and a group.

(2) Determine the size of a subgroup, if the subgroup is to be one-third, or one-fourth of the original group. (respond in like manner if told to find the size of one part if the total number is divided into 3 or 4 equal parts.)

(3) Demonstrate with concrete objects the number of halves, thirds, or fourths, in a unit quantity.

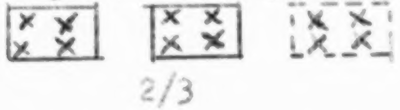
(4) Break up a whole into equal parts so that each part is 1/4 or 1/3 of the unit quantity.

Grade III

(1) Break up a whole into parts so that a part is 1/5, 2/3, 2/4, or 3/4 of a whole.

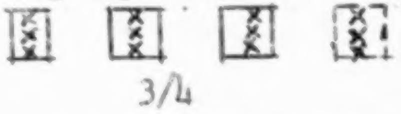
(2) Break up a group of objects (whose cardinal number is a multiple of 3,4, or 5 as the case may be) into groups so that each subgroup is 2/3, 2/4, 3/4, and 1/5 of the original.

12 objects regrouped as:



2/3

12 groups regrouped as:



3/4

(3) Demonstrate with counters the number of fifths in a unit quantity.

(4) Compare one group of objects with another group and determine whether the larger is 2,3, or 4 times the smaller.

(5) Arrange counters or other materials into a given number of equal parts.

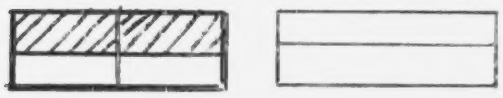
(6) Find 1/2, 1/3, 1/4, 1/5 of a number by dividing respectively 2,3,4, or 5.

Rational Numbers (fractions, decimals, percentage)

- Operations
  - Generalizations

Grade IV

(1) Show by using drawings and fraction devices that taking two parts of a unit divided into fourths is equal to taking one part of the same size unit divided into halves.



2/4 = 1/2

Grade V

(1) Break up one group of objects into several smaller groups. Determine whether each smaller group is 1/8, 1/12, of the original group, or 1/16.

(2) Compare smaller groups of objects with a larger group of objects. Determine whether this smaller group is 1/8, 1/12, or 1/16 the size of the larger group.

(3) Compare a larger group of objects with a smaller group of objects. Determine whether the larger group is 8, 12, or 16 times larger than the smaller group.

(4) Recognize the unit fractional part of a whole becomes smaller as the denominator becomes larger.

(5) Divide more than one unit (3) into different number of equal parts (4) and express the result as a single fraction.

(6) Use the decimal point as a means of locating and fixing the units place which is the starting point of decimal notation.

(7) Explain how decimals are related to the United States monetary system.

(8) Construct diagrams showing place value of tenths and hundredths.

(9) Write a fraction larger or smaller than the one given using the same denominator; then using the same numerator.

(10) Change fractions to higher or lower terms by using pictures, objects, real-life situations, charts, then to abstract computation.

(11) Change mixed numbers to improper fractions by using pictures, objects, real-life situations, charts, then to abstract computation.

(12) Change improper fractions to mixed numbers by using pictures, objects, real-life situations, charts, then to abstract computation.

Rational Numbers (fractions, decimals, percentage)

Operations

Generalizations

Grade V - continued

(13) Change common fractions whose denominators are 10 or multiples of 10 to equivalent decimal fractions.

(14) Change decimal fractions to common fractions, when denominators are 10 or 100.

Grade VI

(1) Write a fraction which is larger or smaller than the one given.
  a. Use the same denominator given
  b. Use the same numerator given
  c. Use unlike denominators

(2) Find the common denominator of three or more unlike fractions.

(3) Reduce fractions to lowest terms by dividing both numerator and denominator by the same number. (Discovery by manipulative device that dividing or multiplying both the numerator and denominator by the same number does not change its value.

(4) Change a decimal fraction to a common fraction and reduce to lowest terms.

(5) Determine by inspection a three-place decimal fraction how many (a) tenths it contains, (b) hundredths it contains, (c) thousandths it contains.

(6) Understand the meaning of percents.

(7) Understand percents = to one, greater than one.

a. $100\% = \frac{100}{100} = 1.00$

b. $200\% = \frac{200}{100} = 2.00$

(8) Understand percents of increases and decreases.

(9) Understand formula: 10% of $n =$

$$\frac{10}{100} \times n =$$

$$.10 \times n$$

(10) Understand repeating and terminating decimals.

(11) Rank and compare decimals according to size.

Rational Numbers (fractions, decimals, percentage)

Operations

Addition

The specific addition skills and understandings regarding rational numbers include the following. The child should be able to:

Grade III

(1) Discover that fractional parts may be put together or added just as whole numbers are added.

2 fourths and 1 fourth = 3 fourths

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

(2) Demonstrate by manipulative devices addition of fractions pertaining to the common fractions.

Grade IV

(1) Add fractions with like and unlike denominators.

(2) Add with decimal point, exchange, rename.
(Exchange means different forms .5 + .3 = .8,
5/10 + 3/10 = 8/10

Grade V

(1) Add a whole number and one or two like fractions by some manipulative device. May also add two or three like fractions by the same method.

(2) Add 2 or 3 mixed numbers with fractions having common denominators (like fractions) by using ruler or some other device.

(3) Add mixed numbers with like fractions.

(4) Add unlike fractions with no denominator larger than 12.

(5) Add fractions not only in vertical position but also in equation form.

(6) Add decimals through hundredths.

Grade VI

(1) Add unlike fractions:

   a. When lowest common denominator is the largest denominator given in the problem -- 2/4 + 1/4 = n

   b. Least common denominator is the product of the denominators -- 1/2 + 1/3 = n

   c. When denominators are factors of least common denominator -- 2/5 + 3/4

   d. When common denominator is not present. (Add by use of concrete objects -- measure on ruler, etc.)

Rational Numbers (fractions, decimals, percentage)

Operations

Addition - continued

Grade VI - continued

(2) Add mixed numbers with unlike fractions including regrouping.

(3) Add fractions and mixed numbers when common denominator is not present.

(4) Add mixed decimals that are carried out to thousandths.

c. Subtraction

Grade III

(1) Discover that fractional parts may be taken away or subtracted just as whole numbers are subtracted.

2 thirds - 1 third = 1 third

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

(2) Demonstrate by manipulative devices subtraction of fractions pertaining to the common fractions.

Grade IV

(1) Subtract fractions with like and unlike denominators.

(2) Subtract with decimal point, renaming, exchange.
(Exchange means different forms)

Grade V

(1) Subtract fractions not only in vertical position but in equation form.

(2) Subtract hundredths with renaming.

Grade VI

(1) Subtract fractions when common denominator is not present.

(2) Subtract with whole and mixed numbers as minuend with unlike fractions in subtrahend and including "regrouping."

(3) Subtract fractions involving all difficulties.

(4) Subtract decimals involving all difficulties.

Rational Numbers (fractions, decimals, percentage)

Operations

Multiplication

Grade III

(1) Use the concept of parts of a whole to parts of a set.

$\frac{1}{5}$ of a set of 20.

Grade IV

(1) Multiply whole numbers by fractions and fraction by whole number.

Grade V

(1) Multiply a whole number by a mixed number using money.

(2) Multiply rational numbers named by fractions and mixed numerals with like and unlike denominators.

(3) Multiply tenths and hundredths.

Grade VI

(1) Multiply decimals involving thousandths.

(2) Multiply a proper fraction by a whole number.

(3) Multiply a whole number by a proper fraction.

(4) Multiply a mixed number by a whole number.

(5) Multiply a whole number by a mixed number.

(6) Multiply a fraction by a fraction.

a. proper fraction by a proper fraction
b. proper fraction by a mixed number
c. mixed number by a mixed number
d. a mixed number by a proper fraction

(7) Recognize that "of" may be interpreted as a multiplication sign in computation.

(8) Multiply decimals - all difficulties.

Rational Numbers (fractions, decimals, percentage)

Operations

Division

Grade V

(1) Divide rational numbers named by fractions and mixed numerals with like and unlike denominators.

(2) Divide decimals - tenths and hundredths.

(3) Find percents.

(4) Find a percent of a number.

Grade VI

(1) Divide a whole number by a fraction -- see it as $2 \div \frac{1}{2} = n$, means how many $\frac{1}{2}$'s are in 2.

(2) Divide a whole number by a mixed number.

(3) Divide a fraction by a fraction, either by changing to a common denominator or by inverting and multiplying.

(4) Operation with decimals to thousandths.

(5) Divide a fraction by a whole number, see it as a problem of dividing a fraction into a number of equal parts.

(6) Divide a mixed number by a fraction.

(7) Divide a mixed number by a mixed number.

(8) Divide a fraction by a mixed number.

(9) Divide a mixed number by a whole number.

Measurement

The specific measurement skills and understandings include the following. The child should be able to:

Grade I

(1) Make simple measures with a foot ruler and yardstick. (Error of measure plus or minus one foot)
(2) Use non-standard units of measure for comparison in the classroom.
(3) Use simple measures of cup, pint, and quart as total units of liquid measure.
(4) Recognize on the clock the time of classroom activities; time for cafeteria, play time, etc.
(5) Tell time when the hands of a clock indicate the hour.
(6) Place hands on a clock face to show any given hour.
(7) Observe the room thermometer and determine whether the room is "too hot"; "too cold," or "a good temperature." (When good room temperature has been indicated on the thermometer by marks.
(8) Use the cent sign (¢) when it is appropriate to do so.
(9) Know the equivalence of coins in amounts to 10¢.
    a. penny equals one cent
    b. nickel equals 5 pennies
    c. five pennies equal one nickel
    d. one dime equals 10 pennies
    e. 10 pennies equals one dime
    f. two nickels equal one dime
    g. one nickel and five pennies equal one dime
(10) Make up story problems involving the use of pennies, nickels, and dimes.
(11) Solve story problems which involves the use of pennies (10), nickels (5), and dimes (10).
(12) Buy and make change in play situations with pennies, nickels, and dimes.
(13) Use calendar to find specific data of the current month.
(14) Copy current data from the calendar as needed.

Grade II

(1) Choose the proper device and/or units for measuring the following quantities:
    a. length - ruler, yardstick, mile, inches, foot, centimeter
    b. liquid - cup, ½ cup, pint, ½ pint, quart, gallon, teaspoon, tablespoon
    c. weight - pounds, ½ pound
    d. temperature - thermometer, degrees, freezing point, above and below zero
    e. time - minutes, hours, days, weeks, months, year, clock
    f. money - quarters, half-dollars, dollars
    g. dry - teaspoon, tablespoon
    h. quantity - dozen, ½ dozen
(2) Tell time when the hands on the clock indicate the hour and half hour.
(3) Use dollar sign ($) when it is appropriate to do so.
(4) Read cent point (.) as "and" when using dollars and cents.
(5) Use cent point (.) when writing dollars and cents.
(6) Use real coins to show equivalence to $1.00.
(7) Tell month and day of birthday.
(8) Use calendar in various ways.

Measurement

Grade III

(1) Write and read the following words as abbreviations and vice versa:

foot	ft.	gallon	gal.
yard	yd.	quart	qt.
inch	in.	pint	pt.
pound	lb.	teaspoon	tsp.
ounce	oz.	tablespoon	tbl.
hour	hr.	minute	min.
year	yr.		

(2) Measure quantities common to everyday experiences.

(3) Measure an object to the nearest unit using appropriate standard measures.

(4) Convert measures by computing or other means.

(5) Tell and record own weight and height.

(6) Tell time when large hand is pointing to 3, 6, 9, or 12, at any time of day.

(7) Make change for a dollar by using a variety of coins and in a variety of ways.

(8) Use calendar in appropriate ways.

(9) Write correctly the names of the months of the year in sequence when names given in scrambled order - (Days of the week also)

(10) Read temperature.

(11) Use number operations involving dollars and cents.

(12) Use fractional parts of measure.

(13) Read charts involving measurement.

(14) Find data involving measurement.

Grade IV

(1) Understand that no measurement is precise since all measurements are approximate measures.

(2) Use time.
  a. Use clock and calendar time.
  b. Understand meaning of a.m. and p.m.
  c. Know the days of the week and the months of the year.

(3) Use measurement units of inch, foot, year, mile, and equivalents.

(4) Use square measures for inches, feet, and yards.

Measurement

Grade IV - continued

(5) Understand and use of cup, pint, quart, and gallon.
(6) Understand and use dozen and half dozen.
(7) Use number operations involving dollars and cents.
(8) Use fractional parts of measures.

Grade V

(1) Add and subtract denominate numbers where a change of unit is not necessary.
(2) Express measurements in different ways as they are needed.
(3) Estimate quantities, check actual measurement.
(4) Illustrate units of measure used in area.
(5) Determine area and perimeter of a rectangle.
(6) Illustrate fractions by using measuring devices.
(7) Recognize measurements to the nearest 1/2, 1/4, 1/16, in. etc.
(8) Determine the best unit of measure to be used in a situation.
(9) Understand the use of the number line.
(10) Understand the use of graphs:
    a. Read and construct bar, line, and circle graphs.
    b. Read picture graphs and tables.
    c. Understand scale drawings.

Grade VI

(1) Understand that the smaller the unit of measure the more accurate the measurement.
(2) Understand that the unit of measure must correspond with what is being measured.
(3) Understand the use of the following time measures:
    a. Time zone, daylight time, standard time
    b. Date line
    c. B.C. and A.D.
    d. Scores of years, decades, centuries
(4) Understand the use of length measures.
    a. English units
    b. Metric units
(5) Understand and use square measure.
(6) Understand and use volume measure.
(7) Understand and use weight.
(8) Understand and use fractional parts of measure represented by fractions and decimal numerals.
(9) Understand and use conversions to linear, square and volume measure.
(10) Understand and use addition, subtraction, multiplication and division with measure.
(11) Understand and use the metric system.
(12) Read and construct bar, line, circle, and picture graphs.
(13) Use maps and scales.
(14) Use tables.
(15) Use estimation.

Geometry (Form, Position, Space)

The specific geometry skills and understandings include the following. The child should be able to:

Grade I

(1) Place objects on an indicated line.

(2) Attach orally the names to circles, squares, and triangles.

(3) Match forms of circles, squares, and triangles when they are seen in drawings, or in the immediate surroundings.

Grade II

(1) Point to north, south, east, and west.

(2) Indicate left and right in reference to oneself.

(3) Trace on a large neighborhood map the route from home to school or school to home and indicate north, south, east, and west.

(4) Use left and right as a position in a situation.
Example: The store is on the left side of the street as you go toward town.

(5) Describe and draw a line, circle, triangle, square and rectangle as forms in descriptions, directions, and basic designs.

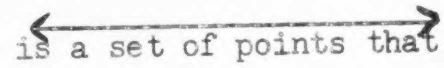
(6) Orally describe a situation, group, or object by using the following.
Follow directions correctly when the following words are used:

here	there	in	the middle
up	down	high	low
over	under	around	across
above	below	out	

Grade III

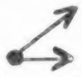
Understand the following geometric concepts:

a. A line segment is a part of a line that has two endpoints.



b. A line is a set of points that go on and on in both directions.

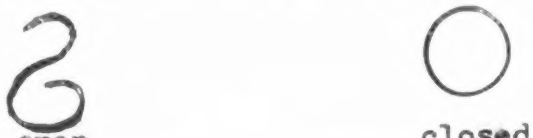
c. A ray is a part of a line that has one endpoint.



d. Angles are made of two rays that have the same endpoint.

e. With closed figures; points inside, outside, and on the figure can be established.

f. There are both open and closed plane figures.



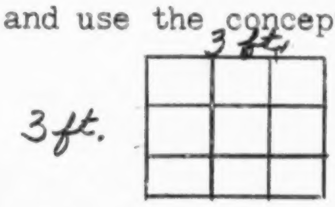
Geometry (Form, Position, Space)

Grade III - continued

(2) Recognize the following forms:

- a. Triangle
- b. Square
- c. Rectangle
- d. Circle
- e. Rectangular prism (cube)

(3) Understand and use the concept of square measure.



3 ft. x 3 ft. = 9 square feet
(surface coverage)

(4) Locate general regions on the map.

(5) Use map legends and scale of miles.

(6) Reproduce in drawings or materials these forms: straight line, circle, triangle, square, rectangle, sphere, and globe.

Grade IV

(1) Develop understanding of points, lines, planes, and angles:

- a. Points are locations and the dot is a symbol for point "."
- b. A line is a set of points. It has infinite length and no endpoints.
- c. A line segment is a part of a line or set of points with two endpoints.
- d. A ray is a part of a line or set of points with one endpoint.
- e. An angle is a set of points in two rays with the same endpoint.

(2) Develop understanding of plane figures and space figures:

- a. Points may be shown inside, outside, and on a closed figure.
- b. Know the definitions of and recognize the following shapes:
  - (1) Triangle
  - (2) Four-sided figures (parallelogram, rhombus, rectangle and square)
  - (3) Circle (radius, center, diameter)
  - (4) Right prisms and cubes
  - (5) Cylinders
  - (6) Spheres

(3) Use measurement in determining the:

- a. linear units for measuring line segments
- b. perimeter of polygons
- c. area of squares and rectangles
- d. surface area of cube

Geometry (Form, Position, Space)

Grade IV - continued

(4) Locate directions north, south, east, and west indicating directions in classrooms, on playground, in community, and on maps.

(5) Give directions in drawing a map of the room, bus route, and community.

(6) Locate directions by the use of a compass.

Grade V

(1) Recognize and understand concepts of points, lines, planes, and angles.
  a. Understand points are locations in space represented by a symbol (.) the dot.
  b. Recognize and describe:
    (1) Line as a set of points
    (2) Space as a set of all points, all locations
    (3) Line segments and endpoints
    (4) Ray as a set of points with one endpoint
    (5) Angle as two rays with common endpoint
    (6) Plane as represented by a table top
  c. Differentiate between:
    (1) Intersecting lines, parallel lines, perpendicular lines
    (2) Perpendicular lines, right angles
    (3) Intersecting lines and angles
    (4) Opposite and supplementary

(2) Recognize closed plane figures:
  a. Understand concept of inside, outside, and on closed figures
  b. Recognize and describe:
    (1) Equilateral triangle
    (2) Four-sided figures: squares, rectangles, parallelograms, and trapezoid
  c. Recognize and describe:
    (1) Circle (central angle, radii, diameter, sector)
    (2) Right prism and cube (edge and faces)
    (3) Pyramid and cone (base and vertex)
    (4) Cylinder
    (5) Sphere (center and radius)

(3) Understand measurement of:
  a. Line segments with linear units
  b. Closed figures (perimeter and area)
    (1) Circle using circumference formula
    (2) Surface area of right prism and cylinder

(4) Construct and copy:
  a. Line segments and angles (labeling)
  b. Perpendicular lines
  c. Angles and bisectors
  d. Congruent lines; polygons; angles -- Congruent figures are the same in shape and size

(5) Show the line of the equator on the globe.

Geometry (Form, Position, Space)

Grade V - continued

(6) Locate meridans, to read longitude and latitude of a point on a globe.

(7) Know the meaning of "elevation" - of buildings, it is the height above the street, or mountains, it is the height above sea level.

Grade VI

(1) Review previous learnings in regards to points, planes, lines and angles.

(2) Recognize and describe angles formed by one or more intersecting lines:
  a. Vertical angles
  b. Corresponding angles
  c. Alternate interior angles

(3) Recognize and describe plane figures, space figures:
  a. Understand 2 dimensional, 3 dimensional figures
  b. Three dimensional figures related to square, rectangle, triangle, and circle
  c. Set of points inside, outside, and on closed figures
  d. Recognize, classify, describe:
    (1) Triangles (right, isosceles, equilateral)
    (2) Quadrilaterals
      a. Parallelograms, rectangle, rhombus, square
      b. Trapezoid
  e. Recognize and describe:
    (1) Circle (central angles)
    (2) Right prism, cube
    (3) Cone, pyramid, right cylinder

(4) Understand and use congruency -- constructing:
  a. Now geometric figures are congruent when they are the same in shape and size
  b. Make figures congruent by copying figures and constructing new figures
  c. Construct and copy:
    (1) Parallel lines and intersecting lines
    (2) Line segments and angles
  d. Determine congruency of:
    (1) Line segments
    (2) Angles
    (3) Triangles
    (4) Polygons

(5) Trace water routes and air routes in a great circle on the globe. To measure the great circle, route and compare its length with that of the parallel of latitudes.

Other Systems and Bases

The specific skills and understandings regarding other systems and bases include the following. The child should be able to:

Grade II

(1) Understand and use Roman numerals to 10 (optional)

Grade III

(1) Understand and use Roman numerals through 39.

Grade IV

(1) Review Roman numerals through 39.

(2) Understand and use Roman numerals through "C" (100)

Grade V

(1) Understand and use Roman numerals through "D" and "M".

(2) Recognize the importance of the lack of a symbol for zero with Roman numerals.

(3) Understand "Base 5" (optional)
(This skill is undertaken so that of structure of the base 10 system can be better understood)
   a. Count by groups of 5
   b. Convert from base 10 to base 5

Grade VI

(1) Understand and use Roman numerals through "M" (1000)

(2) Understand the additive and subtractive principles involved in the use of Roman numerals.

(3) Understand the Mayan System and compare to Hindu-Arabic System.

(4) Understand Base 8.

   a. Understand grouping principle
   b. Understand the use of digits and their use
   c. Count on base eight
   d. Add and subtract on base eight

Problem Solving

The specific skills and understandings regarding problem solving include the following. The child should be able to:

Grade I

(1) Develop oral and written word problems about children in the class and other natural situtations that require simple addition or subtraction for their solution. Include problems dealing with money and measurement.

Grade II

(1) Develop skill in problem analysis.

(2) Work with problems involving money, measures, map reading, fractions, and/or addition and subtraction of whole numbers.

Grade III

(1) Originate problems.

(2) Use problem solving analysis to develop independence.

(3) Solve problems involving measures, money, fractions, and operations on whole numbers.

Grade IV

(1) Write number sentences for problems.

(2) Solve problems dealing with various measures and operations.

Grade V

(1) Solve two-step problems.

(2) Write number sentences for problems.

(3) Solve problems dealing with various measures and operations.

(4) Practice in the use of inductive reasoning.
(Drawing general conclusions from several known, specific cases)

(5) Practice on questions calling for deductive reasoning.
(Reasoning from general to specific)

Grade VI

(1) Solve wide variety of problems.

(2) Continue practice in problems dealing with inductive and deductive reasoning.

Instructional Materials

Introduction

Some boys and girls succeed in arithmetic easily while others have stomach ache, headache, and all sorts of pains over it. WHY?

We believe that all children can learn arithmetic interestingly and enthusiastically -- if they learn it meaningfully. Learning arithmetic meaningfully is simply having all children take their "first steps" in a new concept through the use of concrete things -- things to hold in their hands, things to count, things to divide. Primary teachers who have collections of "bottle caps 'n scraps" for pupils to touch and handle are approaching arithmetic meaningfully. Such collections may include sticks, beads, jacks, clothespins, spools, buttons, macaroni shells, plastic scraps, egg cartons with paper mache eggs, curtain rings, and bottle tops. "Counters," made from coat hangers, with 10 beads for first graders, can be provided for each child. Girls can be invited to donate their old pop beads which will help the process of learning to count by twos, threes, fours, and so on when children actually arrange the colors through manipulation.

One teacher declares that her children have learned fractions easily and successfully since she began by letting them "eat the fractions." She places several apples on her desk and asks the children to plan how to divide them so that each will get an equal share. She finds that a pan of fudge offers experiences in developing two concepts -- square measure as well as fractional parts.

We remember the old proverb:

> "What I hear, I soon forget;
> What I see, I may remember;
> But what I do, I know."

We know that children say, "Wait a minute," "I have a hundred like that at home," or "Give me the biggest half" without a clear understanding of the meaning. We know that verbal explanations, alone, are not enough. First hand experiences must be arranged through the use of concrete, manipulative materials to help children acquire clear concepts of number, size, and quantity. The materials should be used and re-used as needed until the "visualization" step is reached. "Visualization" is the beginning of abstract thinking. Then it is just as important that we follow through to the abstract as it is that we begin with the concrete. We must know that when a child understands, he must be guided to begin to think in the abstract (visualize) until he finds no further need for concrete things for that particular arithmetic concept. The concrete things must be kept on hand, however, for those who still need them. The steps through which a first grade child should progress from concrete to abstract numbers are as follows:

1. The concrete -- Pupils work with objects which they can pick up, handle, and move about.

2. The picture -- Pupils work with pictures of toys, animals, people, and other objects.

3. The semi-concrete -- Pupils work with dots, circles, triangles, and other forms.

4. The abstract -- Pupils work with the number symbols 1,2,3, and so on, without any aids to indicate their meanings.

Please Note: The preceding does not mean that a child goes from the concrete to the abstract, directly, and stays there. It is a back and forth process. After the work is started, the teacher may use all steps in one day to compare and review.

What about drill? Drill is necessary. We drill for automatic response after the child understands and is beginning to think in the abstract.

We hope that every teacher will make a meaningful approach in the teaching of arithmetic. This will require some concrete, manipulative things and many visual aids. See the following suggestions:

Manipulative Materials

Teachers should introduce every new concept in arithmetic in a concrete way. Concrete manipulative materials should be on hand in every classroom to be used by all pupils when they are taking first steps toward a new learning; and the materials should be used and re-used as needed until each child reaches the "visualization step" which is thinking in the abstract. Manipulative materials may be "home made" collections of items which children can handle, or they may be purchased.

Suggestions for "home made" materials are listed below:

1. Things to Count

   Plastic squares - made from scrap plastic (30 cents a pound)
   Plastic straws - (10 cents per package)
   Bottle caps
   Pop beads
   Macaroni shells
   Tongue depressors
   Sticks
   Buttons
   Clothespins
   Jacks
   Curtain rings
   Spools

2. "From Bottle Tops 'n Scraps" - a folder containing a reprint from the Indiana Teacher. (For kindergarten and first grade teachers)

3. Calendar to Record Weather - (For kindergarten and first grade)

   Many values are involved:
      Develops concept of month, week, day of the week, season, and year
      Stimulates observation
      Correlates with science

4. Temperature Graph

   For all grades - the weather chairman marks the 8:30 A.M. temperature each day.

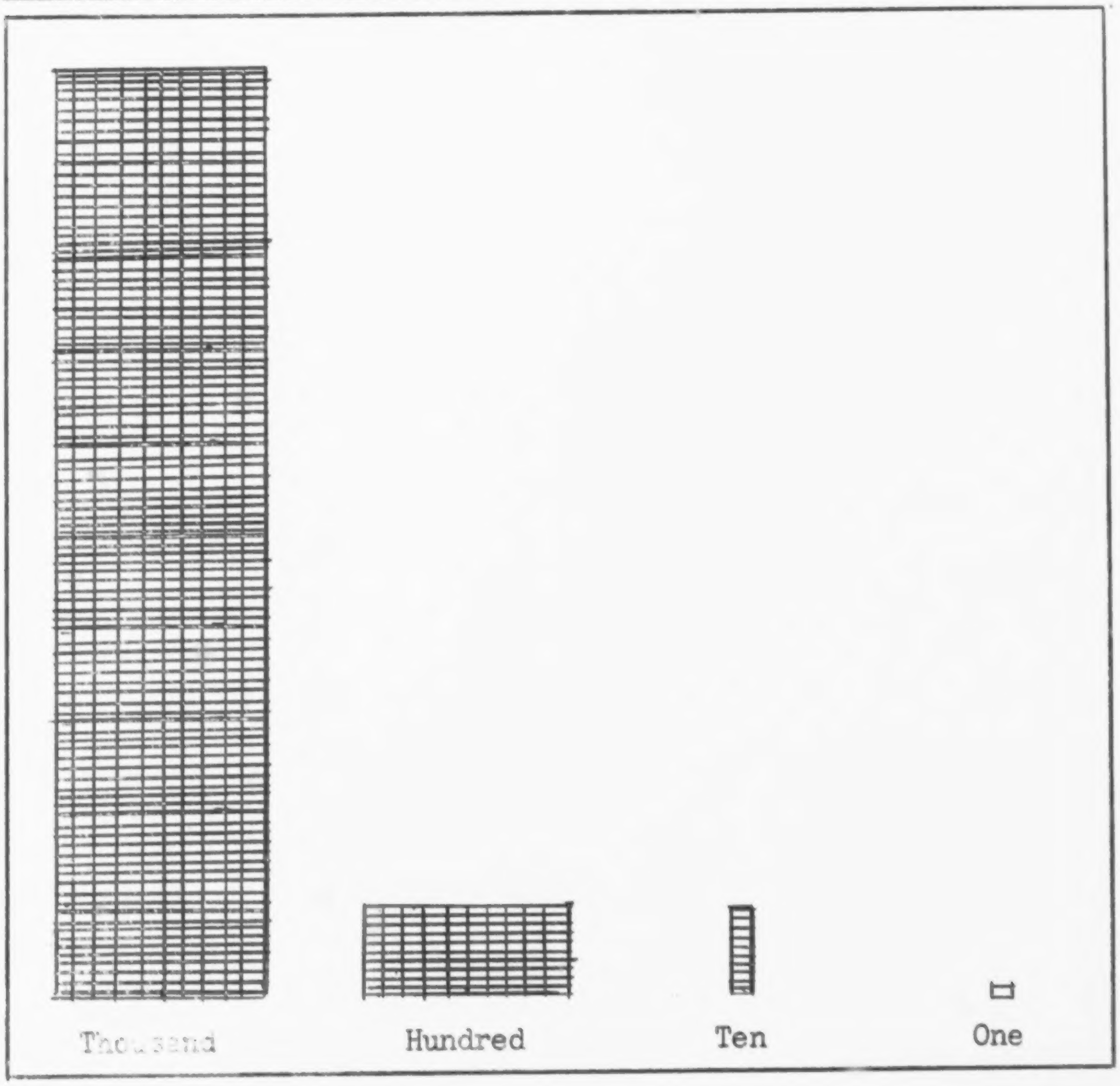
5. Home Made Thermometer

   For kindergarten and first grade

6. Real Thermometer

For grades 2 through 6. Place them outside a window.

7. The Value of One in Each Position in Our Numbers (Enlarge for poster)



8. Number Ladder (Can be used for many purposes)

Counting by ones, twos, threes, and so on
Recording pupils' heights
Showing inch, foot, and yard measurements

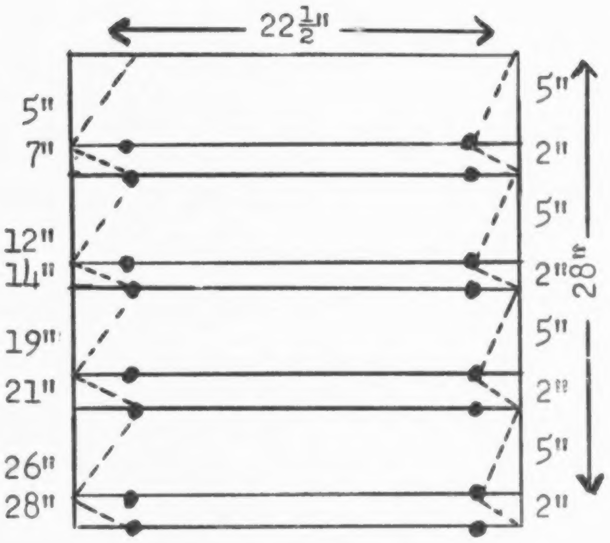
9. Classroom Weight Chart

Free (Write to: Borden's Milk Company, Chicago, Illinois)

10. Place Value Chart

Made from one sheet of tagboard, stapler, and black tape.

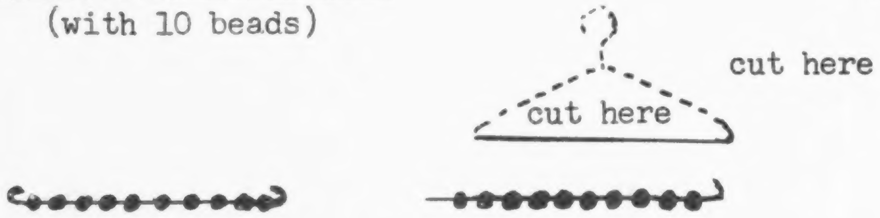
Use one sheet of tagboard $22\frac{1}{2}$" x 28". Start from the bottom and mark dots in pairs. Mark so that dots in a pair are 2 inches apart and pairs are 5 inches apart. There will be 5 inches from the last dot to the top of the tagboard.



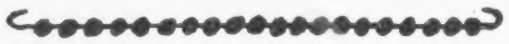
Fold tagboard, connecting dots on each side. Reverse the top fold of each of three pairs to form pockets two inches deep. Fold bottom 2 inches upward. Be sure pockets are open at top.

Bind with a gummed tape. Staples can be used to divide the ones', tens', and hundreds' places.

11. Counter for First Grade
(with 10 beads)

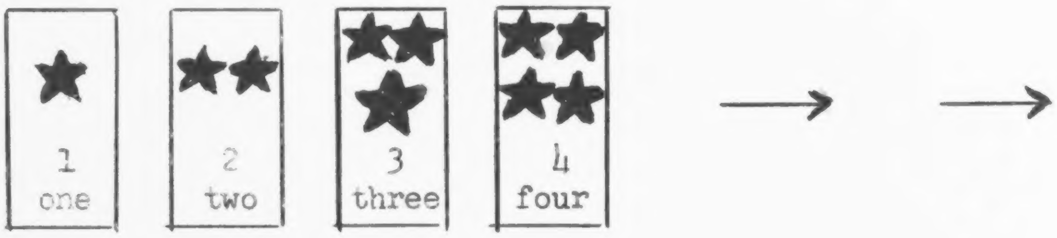


12. Counter for Second Grade
(with 20 beads)



Same as above, using 20 beads

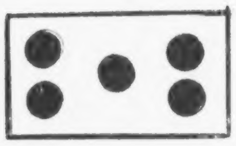
13. Picture, Number, and Number Name Cards
(from 1 through 10 for 1st grade)
(from 1 through 20 for 2nd grade)



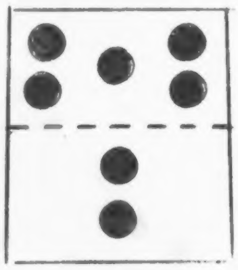
14. Large Wooden Numbers for First Grade Children Who Need Much Help

15. Sandpaper Letters
Large letters made from sandpaper and pasted on a card offer help through the kinesthetic approach to first grade children who have difficulty with number concepts.

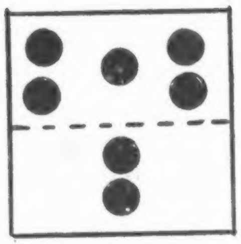
16. Folding Perception Cards



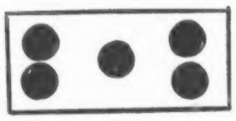
You see $\underline{\quad 5 \quad}$



Open and see $\begin{array}{r}5\\+2\\\hline 7\end{array}$



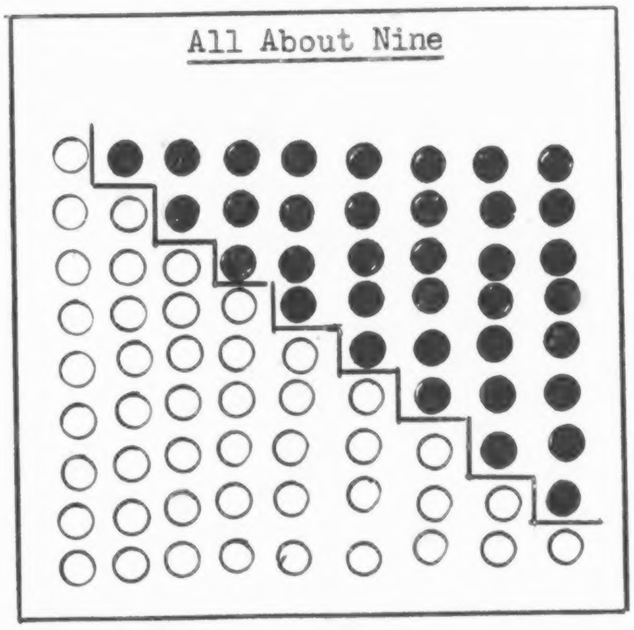
You see 7 in all.



Close to show $\begin{array}{r}7\\-2\\\hline 5\end{array}$

17. "All About" Charts
(See sample below)

All About Nine Chart

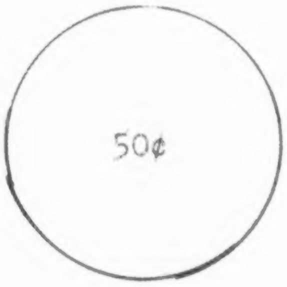


1 + 8 = 9
2 + 7 = 9
3 + 6 = 9
4 + 5 = 9
5 + 4 = 9
6 + 3 = 9
7 + 2 = 9
8 + 1 = 9
9 + 0 = 9
9 - 1 = 8
9 - 2 = 7
9 - 3 = 6
9 - 4 = 5
9 - 5 = 4
9 - 6 = 3
9 - 8 = 1
9 - 8 = 1

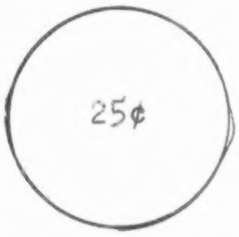
9 - 0 = 9
8 + 1 = 9
7 + 2 = 9
6 + 3 = 9
5 + 4 = 9
4 + 5 = 9
3 + 6 = 9
2 + 7 = 9
1 + 8 = 9
9 - 8 = 1
9 - 7 = 2
9 - 6 = 3
9 - 5 = 4
9 - 4 = 5
9 - 3 = 6
9 - 2 = 7
9 - 1 = 8

18. Toy Money (Use in all grades as needed)
Have middle grade pupils make from cardboard.

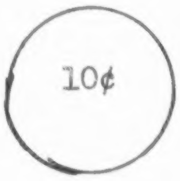
On other side, write
One Dollar



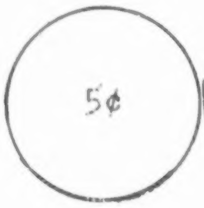
Fifty Cents



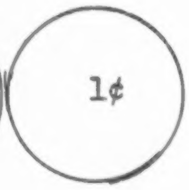
Twenty-five
Cents



Ten
Cents



Five
Cents



One
Cent

If you purchase toy money, we recommend the cardboard type. The metal money cannot be identified easily.

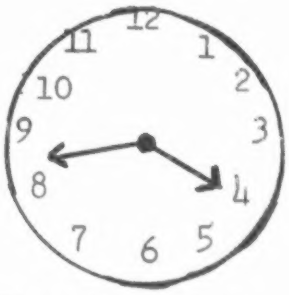
One Dollar

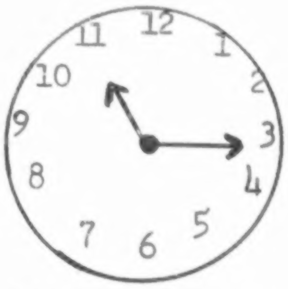
Bills can be made from mineograph paper.

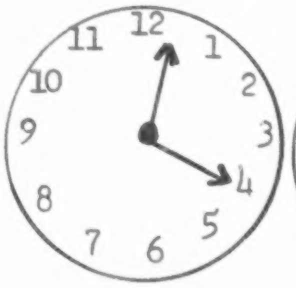
19. Bulletin Board Clock Faces

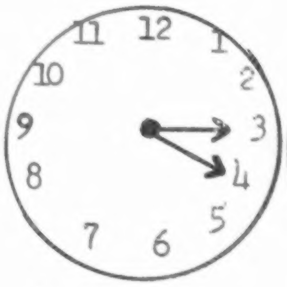
(For kindergarten and first grade)

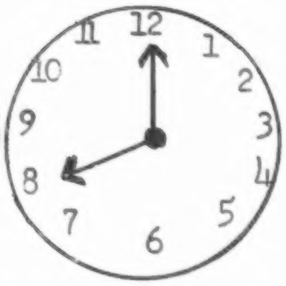
To visualize a daily schedule
To motivate planning
To stimulate time telling
To suggest proper bedtime


Time for School


Time for Lunch


Time for School after Lunch


Time to go Home


Time for Bed

20. Clock Faces - to master time telling

One for each pupil - grades 2-3

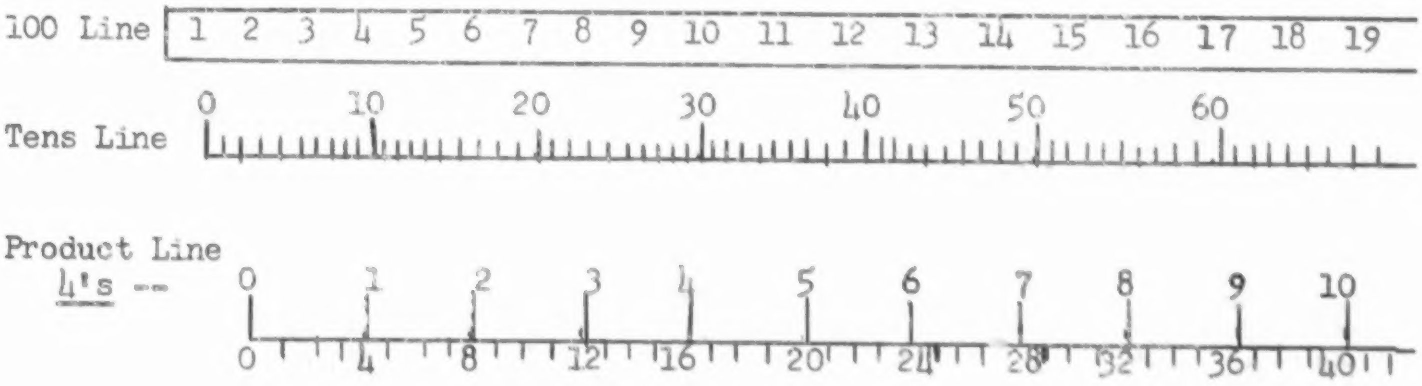
Make from - a paper plate
black cardboard
red crayon
paper fastener
one inch of plastic tubing

21. Pop Beads (Different colors)

Arrange by twos, threes, fours, and so on.

Make a hundred line, using a bead of a different color for 10, 20, 30 - - -

22. Number Lines


100 Line
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
Tens Line
0 10 20 30 40 50 60
Product Line
4's --
0 1 2 3 4 5 6 7 8 9 10
0 4 8 12 16 20 24 28 32 36 40

23. Measures

Measuring Cups: one
one-half
one-third
one-fourth

Gallon (Ask your cafeteria manager for a gallon can)

Half-gallon

Quart

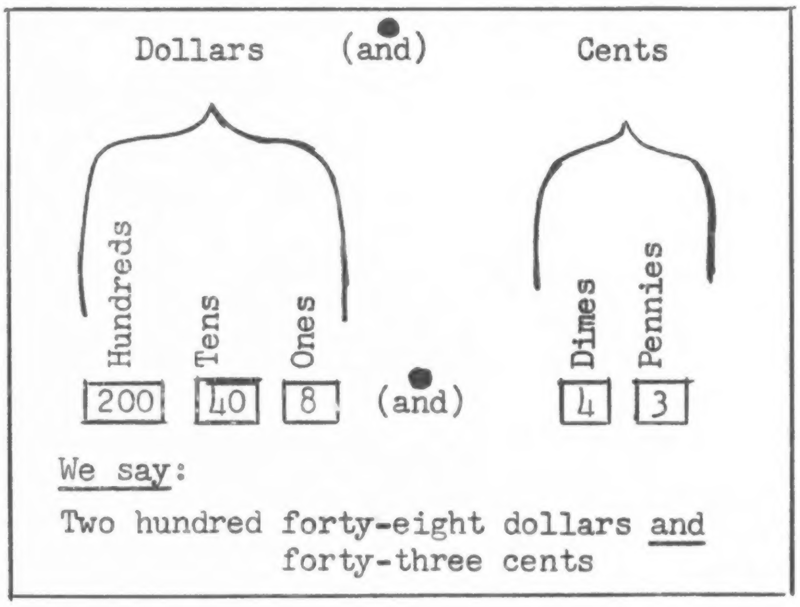
Pint

Half-pint

24. Tape Measure - Use one-inch gummed paper. Fold over and stick together so that a tape measure of one-half inch width can be made. After pupils mark off and number the inches, they will know an inch. Mark fractional parts of an inch as needed.

25. Foot - Yard - Rod -- Use a rod of plastic clothesline. Paint a foot of it one color. Mark the yards.

26. Dollars and Cents Chart



We say:

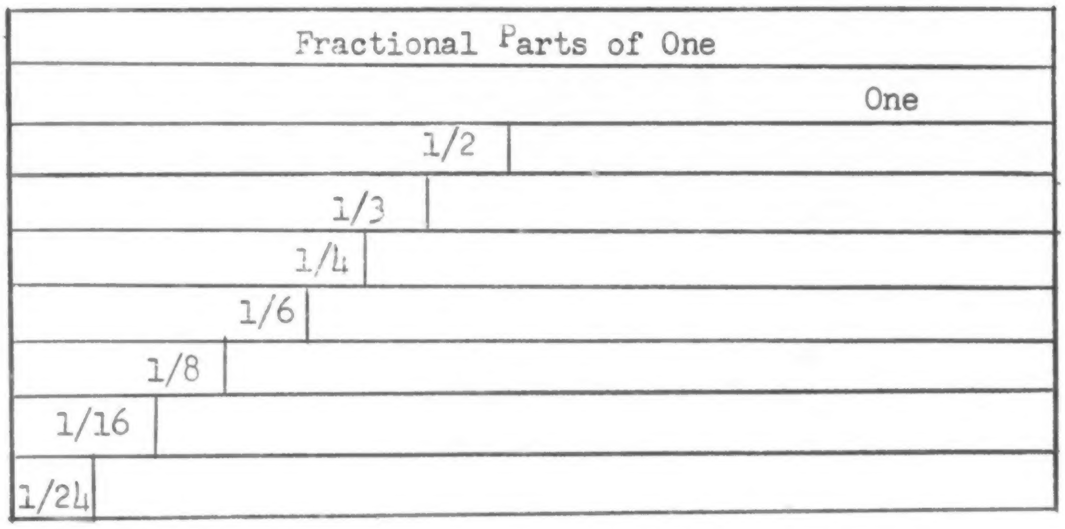
Two hundred forty-eight dollars and forty-three cents

27. Tens Square

Use as a tens square in primary grades.

Use as a fraction board in middle grades.

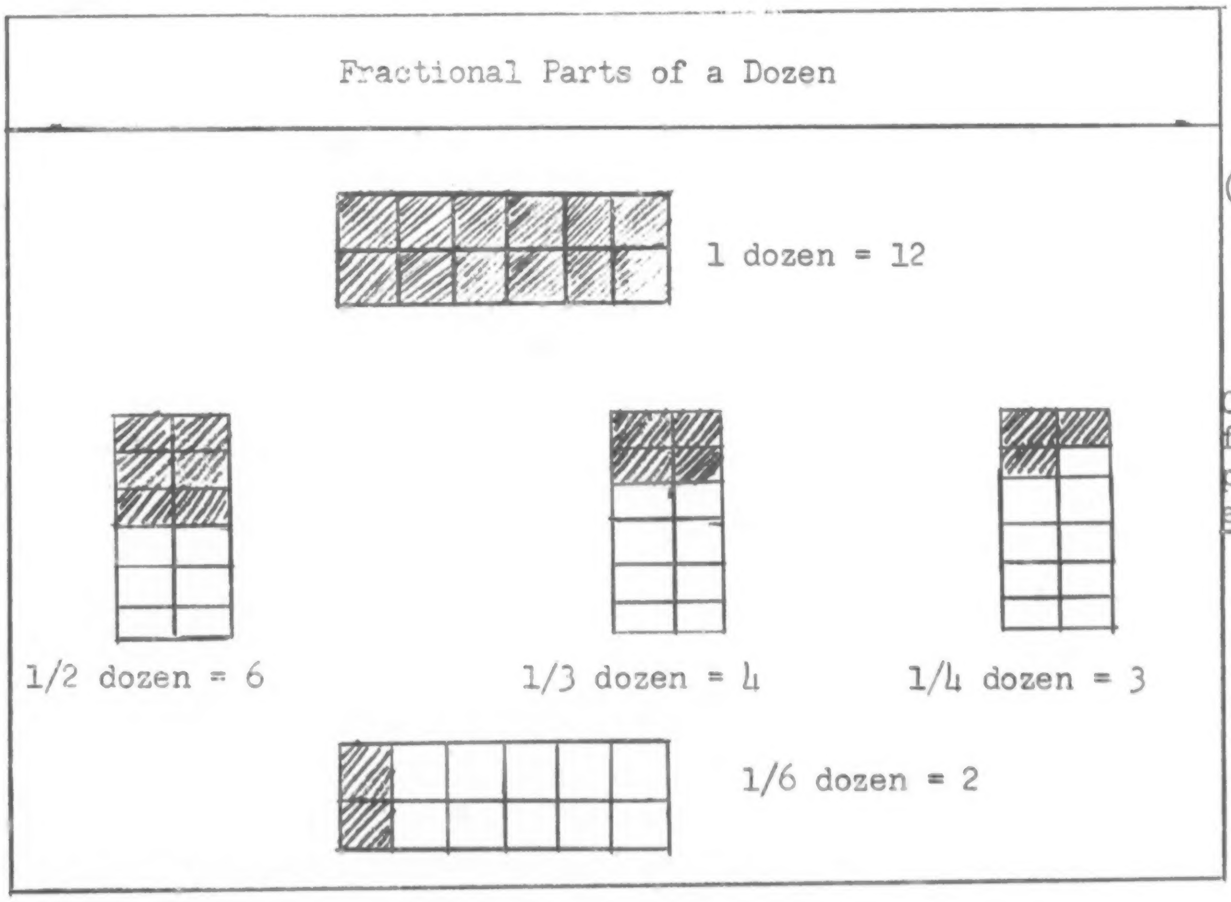
28. Fractional Parts of One (Chart)


Fractional Parts of One
One
1/2
1/3
1/4
1/6
1/8
1/16
1/24

29. Fractional Parts of One (Pocket)

Every pupil who is working with fractions should have one. Can be made by 6th graders. Use construction paper - make fraction wheels. Cut so that there are many halves, thirds, fourths, sixths, eighths, and twelfths.

30. Fractional Parts of a Dozen (Poster)

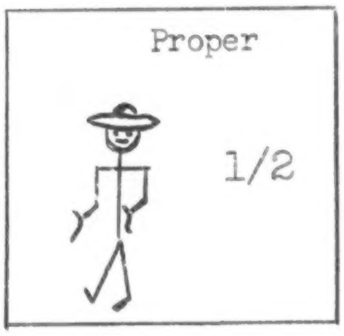

Fractional Parts of a Dozen
1 dozen = 12
1/2 dozen = 6
1/3 dozen = 4
1/4 dozen = 3
1/6 dozen = 2

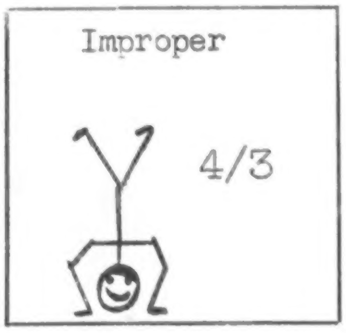
Egg Carton
(all in one
color)

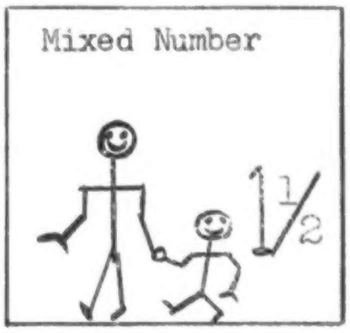
Color each
fractional
part a diff-
erent color

Papier maché eggs may be made, also.

31. Aids to Identifying Proper Fractions, Improper Fractions, and Mixed Numbers - (Bulletin Board Pictures)







32. Table of Least Common Denominators

Make a large chart (See text)

33. Free - Mathematical Words Chart

Scott, Foresman and Company
433 East Erie Street
Chicago, Illinois

34. Free - How to Read Large Numbers Chart

Scott, Foresman and Company
433 East Erie Street
Chicago, Illinois

35. Free - Large Addition Facts Chart
Large Subtraction Facts Chart
Large Multiplication Facts Chart
Large Division Facts Chart

D. C, Heath and Company
1815 Prairie Avenue
Chicago 16, Illinois

36. Inch Cubes of Wood

Construction:

Cut several hundred 1" cubes in the school shop and enamel, varnish, or shellac to make cleaning easier. Or purchase the cubes from any school supply house if shop facilities are not available.

Sugar cubes and starch cubes may also be used to build small solids.

Use:

To build rectangular solids of various sizes and illustrate volume by showing that they contain as many cubic units as there are units in one layer times the number of layers.

INSTRUCTIONAL MATERIALS

Books for Children

1. Kindergarten - Grade 3

Adler, Irving. Numerals: New Dresses for Old Numbers.
John Day, 1964.

Newest ways of counting. Excellent treatment of place value and systems in bases other than ten.

Auerbach, Majorie. Seven Uncles Come for Dinner. A.A. Knopf, 1963.

Emile's shopping tour for seven uncles provides an interesting story.

Baer, Howard. Now This, Now That. Holiday House, 1957.

Book which begins the probing of probability. Encourages children to see things from distances and angles to develop concepts of big, little, up, down, taller, straight, under, on, far, and near.

Barr, Catherine. Seven Chicks Missing. Henry Z. Walck, 1962.

Story of a mother goose who loses one chick after another while taking her brood walking.

Baum, Arline. One Bright Monday. Random House, 1962.

Rhythmic prose helps pre-schoolers and primary children learn the days of the week as well as their numbers.

Behn, Harry. All Kinds of Time. Harcourt, Brace and Co., 1950.

Poetic book about clocks, time, and the seasons.

Beim, Jerrold. The Smallest Boy in Class. Wm. R. Morrow, 1949.

Terms such as big, biggest, small, smallest, tiny, tall, and tallest are used to develop concepts of comparison through the antics of the smallest boy in class.

Bendick, Jeanne. All Around You. McGraw-Hill Book Company, 1951.

Science picture book describes the why and how of the world around us with simple pictures and vocabulary of comparison, number, and measurement.

______________. Take A Number. Whittlesey House, 1961.

Facts, ideas, and puzzles are presented to explain numbers from finger counting to computers.

Berkley, Ethel. Big and Little, Up and Down; Early Concepts of Size and Direction. Wm. R. Scott, 1960.

Valuable for developing concepts in size and direction. Should develop beginnings of mathematical vocabulary: big little, long, tall, wide, narrow, short, up, down, high, low, under, over, top, bottom.

Bianco, Pamela. Doll in the Window. Henry Z. Walck, 1953.

Saving pennies to buy Christmas presents for her five sisters, a little girl has to decide how much to save to buy each gift and how much she can spend for each with the dollar she has saved.

Bishop, Claire. Five Chinese Brothers. Coward-McCann, 1938.

Useful in developing ideas of cardinal and ordinal numbers.

________________. Twenty-two Bears. Viking Press, 1964.

Story of bears that come out of everywhere makes for fun in developing concepts of numbers in groups from 1 to 22 for young children. Clever illustrations.

Blough, Glenn. Wait For The Sunshine. McGraw-Hill Book Company, 1954.

Story of seasons and growing things, uses many ideas of measurement and comparison as well as cardinal and ordinal numbers.

Branley, Franklyn. Big Tracks, Little Tracks. Crowell, 1960.

Size and comparison are shown through footprints or tracks of insects, animals, birds, and human beings. Cleary illustrated.

Brann, Esther. Five Puppies For Sale. Macmillian Company, 1948.

Ideas of numbers one through six, addition, subtraction, cents, and other arithmetic concepts are explained through a little girl's story of a family of puppies.

Brenner, Barbara. The Five Pennies. A. A. Knopf, 1963.

Micky learns the value of money in a subtle introduction to arithmetic. He takes five pennies to buy an animal.

Brindze, Ruth. Story of Our Calendar. Vanguard Press, 1949.

Explains understanding a calendar, tells of calendars through the ages and how people kept account of time by stars, moon, etc.

Budney, Blossom. A Cat Can't Count. Lothrop, Lee, and Shepard, 1962.

This book helps the young child count through word and number associations.

____________. A Kiss Is Round. Lothrop, Lee, and Shepard, 1954.

Picture verse book tells of familiar objects which are round. Stimulation for the child's perception of forms.

Chalmers, Audrey, Hundreds and Hundreds of Pancakes, Viking Press, 1942.

Humorous story about large numbers of pancakes eaten by a family. Ideas of size, position, comparison, and measurement are conveyed.

Colman, Hila. Watch That Watch. Wm. R. Morrow, 1962.

A "fun" story useful for impressing the child with the importance of time in everyday life.

Cooke, David C. How Money is Made. Dodd, Mead and Company, 1962.

Meaningful photography accompanies text of currency and coin manufacture from design to wrapping. Tells of disposal of old money and recognition of counterfeit.

Courtney, William. What Does A Barometer Do? Little, Brown and Company, 1963.

This book answers many questions about barometers. Uses mathematical concepts of measurement.

D'Aulaire, Ingri. Don't Count Your Chicks. Doubleday and Company, 1943.

An old lady counts eggs and money and makes use of one-to-one idea while dreaming of more chickens, eggs, and money.

Duvoisin, Roger. Two Lonely Ducks. A. A. Knopf, 1955.

Provides lessons in numbers by counting ducklings, days of the week, weeks in the month. Ordinal numbers also presented.

Elkin, Benjamin. Six Foolish Fisherman. Children's Press, 1957.

Six fishing brothers dear one of them is missing because in counting, each forgot to count himself. Provides meaningful counting experiences.

Emberley, Ed. The Wing On a Flea. Little, Brown and Company, 1961.

A book to develop concepts about all kinds of shapes. Small amount of text on a page of color and good illustrations.

Everson, Dale. Mrs. Popover Goes to the Zoo. Wm. R. Morrow, 1963.

Mrs. Popover loses 24 children and then finds them again in this amusing, illustrated counting book.

Federico, Helen. Numbers. Golden Press, 1963.

Big outline numbers are filled with objects to count.

Feravolo, Rocco. Wonders of Mathematics. Dodd, Mead and Company, 1963.

Includes simple activities and problems covering development of number systems and mathematics.

Fisher, Margery M. One and One. Dial Press, 1963.

An introduction for the young child to the basic theory of numbers. Not a counting book but a supplement to the study of what numbers are and how they work.

Francoise. What Time Is It, Jeanne-Marie? Charles Scribner's Sons, 1963.

Following Jeanne-Marie through the hours of a fun-filled day makes learning to tell time a delightful lesson.

Friskey, Margaret. Chicken Little, Count to Ten. Children's Press, 1946.

Ideas of numbers one to ten, explained as a chicken who doesn't know how to drink water, meets 1 cow, 2 elephants, 3 camels, etc.

______________. The Mystery of the Farmer's Three Fives. Children's Press, 1963.

Presents a new approach to number concepts of "few" and "many" and relative size groupings by using barnyard animals as the units.

Gag, Wanda. Millions of Cats. Coward-McCann, 1945.

Story includes situations helpful to explain ideas of around, over, through, hundreds, thousands, millions, etc. Provides children with readiness for large numbers.

Grayson, Marion F. Let's Do Fingerplays. Robert Luce, 1962.

Fine collection of fingerplays will teach small children their numbers and how to count. Other activities can grow out of these action rhymes as well.

Hoberman, Mary Ann. All My Shoes Come in Twos. Little, Brown and Company, 1957.

Rhyming story tells of all kinds of shoes and uses idea of "two" over and over again.

Jeruchim, Cecile. Hello! Do You Know My Name? G. P. Putnam's Sons, 1963.

Simple rhymes about shapes of familiar objects introduce young children to a line and its various forms.

Kay, Helen. One Mitten Lewis. Lothrop, Lee, and Shepard, 1955.

Humorous story of a little boy who always loses one mitten. Use of words: pair, both, one, each, first, next.

Kepes, Juliet. Two Little Birds and Three. Houghton Mifflin, 1960.

Simple text and clever illustrations show beginning number concepts in groups of 2, 3, and 5.

Kessler, Leonard. I Made a Line. Grosset and Dunlap, 1962.

Story rich in concepts of "round" and "square"; shows basic line used in drawing.

____________. The Worm, The Bird, and You. Dodd, Mead and Company, 1962.

Picture book presents concepts of perspective by showing that size is relative. Bold color used.

Krasilovsky, Phyllis. The Very Little Boy. Doubleday, 1962.

Little boy one day finds he has grown large enough to eliminate many handicaps of being too small. Concepts of relative size and growth are presented.

____________. The Very Little Girl. Doubleday, 1953.

Story of little girl smaller than rose bush, kitchen stool, her mother's work basket. As she grew, she could reach fish bowl, see over garden fence. Pictures depict ratio and proportion.

Krauss, Ruth. The Growing Story. Harper Brothers, 1947.

Little boy watches flowers and animals grow all summer, but doesn't realize until fall that he, too, has grown.

Langstaff, John. Over in the Meadow. Harcourt, Brace and World, 1957.

Picture book based on old animal counting song. Groups of 1 to 10 shown realistically.

Lansdown, Brenda. Arithmetic for Beginners. Grosset and Dunlap, 1959.

Many illustrations of everyday things help a child recognize groups from 1 to 10.

Leodhas, Sorche Nic. All in the Morning Early. Holt, Rinehart and Winston, 1963.

Picture book in rhyme and prose provides children with concepts of groups of things: 2 ewes, 3 gypsies, 4 farmers, etc.

Lewell, John. True Book of Airports and Airplanes. Grosset and Dunlap, 1956.

Language of number, position, comparison, and measurement appears in this simple, factual book about airports.

Lionni, Leo. Inch by Inch. Obolensky, 1960.

Picture book with pictures of how an inchworm proves his usefulness as a "measurer."

LITTLE OWL Reading Time Series. Holt, Rinehart and Winston, 1963.

Considine, Kate. One, Two, Three, Four.

Basic concept to associate cardinal and ordinal numbers, one through twelve, is provided with the twelve months of the year.

Fehr, Howard F. Five is 5.

Different meanings as well as ways of expressing 5 are shown through groups of objects, and then as a symbol. Skill in picture reading, word and number sentences is developed.

__________. This is my Family.

This book deals with the mathematical concepts of sets, and subsets. the word "set" is only conveyed through concepts, not specifically used.

Hall, William. Captain Murphy's Tugboats.

Basic addition process is studied, and skill in reading number sentences is developed through visual and symbolic statements of number.

Heller, Aaron. Let's Take a Walk.

Use of ordinal numbers on houses, stores, and street signs provides experience in discussing number concepts, numbers progress along the street to show direction children take in their walk.

Otto, Margaret, Three Little Dachshunds.

The concept "three" is developed in a simple manner in rhythmic story and pictures.

Quackenbush, Robert. Poems for Counting.

Counting rhymes are presented here to give experience in counting from 1 to 10, and in reading number symbols and number words.

Sullivan, Joan. Round is a Pancake.

Concept of "roundness" presented through familiar and unfamiliar things. Subtle background of shapes in the form of squares, triangles, and rectangles.

Wing, Henry R. Ten Pennies for Candy.

One-to-one relationship in the study of arithmetic; number terms which match pennies with children and children with candy are developed.

___________. What is Big?

Concept of "big" and "small" is shown through colorful illustrations. Discussion will lead children to explore further ideas of size.

Wittram, H. R. Going Up, Going Down.

Introduces the number line, positive and negative numbers, and provides counting experiences.

McCullough, John. Farther and Faster. Crowell, 1954.

Vocabulary of comparison is employed extensively, resulting from showing how better means of transportation comes about through man's desire to travel farther and faster.

McLeod, Emilie. One Snail and Me. Little, Brown and Company, 1961.

Story about a little girl and a bathtub full of animals waiting to be counted. Colorful illustrations.

Malter, Morton S. Our Largest Animals. Albert Whitman, 1958.

Language of relative size, weight and time, and many number names from 1 to 12,000 are effectively used as largest animals of the world are described.

___________. Our Tiniest Animals. Albert Whitman, 1955.

Language of number, weight, time, and relative size are effectively used as small animal life is described.

Marino, Dorothy. Edward and the Boxes. J. B. Lippincott, 1957.

Concept of comparison is applied.

Massoglia, Elinor. Fun-time Paper Folding. Children's Press, 1959.

An understanding of elementary language of comparison and shape is needed to make objects without cutting or pasting.

Memling, Carl. I Can Count. Golden Press, 1963.

Number concepts developed through small and large animals in groups with numerals, 1 to 10.

Montresor, Beni. House of Flowers, House of Stars. A. A. Knopf, 1962.

Concept of shapes developed through colorful drawings and words: plump, skinny, lanky, squat. Different kinds of houses, such as mouse house, flower house, star house.

Moore, Lilian. My Big Golden Counting Book. Simon and Schuster, 1962 rev. ed.

Large animal pictures, simple language, and number symbols are used to illustrate and describe groups from 1 through 10.

Myller, Rolf. How Big is a Foot? Atheneum Press, 1962.

Nonsense story pointing up need for standard measurement, an important mathematical concept.

Pine, Tillie S. The Chinese Knew. McGraw-Hill Book Company, 1958.

Ancient Chinese employed the same scientific principles as used today in making and using abacus, water clocks, and other activities.

Rand, Ann. Little I. Harcourt, Brace and World, 1962.

"Little I" has a lonely life until he enters different groups and relationships with other figures to produce new arithmetical concepts.

Ravielli, Anthony. The World Is Round. Viking Press, 1963.

Pictorial account of why the earth seems flat to us and how man learned its true shape.

Rice, Stanley. Tell Time. Harcourt, Brace and World, 1963.

Programed instruction for individual student with teacher help. Introduces many ways of telling time.

Russell, Betty. Big Store -- Funny Door. Albert Whitman, 1955.

Two children travel through a supermarket, using ideas of number, operation, and measurement.

Schatz, Letta. When Will My Birthday Be? McGraw-Hill Book Company, 1962.

Concept of time is developed when Benjy must wait through summer, fall, and winter until spring to be one year older.

Schlein, Miriam. City Boy, Country Boy. Grosset and Dunlap, 1955.

A boy living in the city and one living in the country describe their lives during one year, using concepts of time, size, position, and numbers.

__________. Fast Is Not A Ladybug. Wm. R. Scott, 1953.

Narrative about moving objects helps to make concepts of "fast" and "slow" meaningful.

__________. Heavy Is A Hippopotamus. Wm. R. Scott, 1954.

Development of an understanding of weights, and measures by showing different ways of thinking in relation to some familiar objects.

__________. It's About Time. Wm. R. Scott, 1955.

Concepts of time in relation to seconds, minutes, hours, days, seasons, etc., are explained. Verse and pictures develop concept of time.

__________. Shapes. Wm. R. Scott, 1962.

Children are made aware of concepts of form and design as well as the utility of shapes. Use made of words "round," "line," "curve," "long," and "tall."

Schneider, Herman. How Big is Big? Wm. R. Scott, 1946.

Presentation of clearer concept of what is big, how big is little. Relative size understanding through comparison of an elephant, tree, skyscraper, and the earth. Smallness demonstrated in same way with dog, mouse, flea, algac, atoms, electrons.

Seuss, Dr. Dr. McFligot's Pool. Random House, 1947.

Ideas of number, comparison, shape, size, length, and height are used.

__________. One Fish, Two Fish, Red Fish, Blue Fish. Random House, 1960.

Numbers are introduced through groups of the usual Seuss characters accompanied by rhyming nonsense text.

Shapp, Charles. Let's Find Out What's Big and What's Small. Franklin Watts, 1959.

Text and pictures show clearly the relative meaning of size.

Shapp, Martha. Let's Find Out What's Light and What's Heavy. Franklin Watts, 1961.

Simple experiments designed to show size.

Sharp, Elizabeth. Simple Machines and How They Work. Random House, 1959.

An account of wheels, pulleys, levers, screws, wedges, and inclined planes makes wide use of the language of size, position, and comparison.

Thaler, Mike. Penny Pencil. Harper and Row, 1963.

Adventures of a pencil as she grows smaller and smaller. Will provide good reading for children as they grow larger and larger.

Tudor, Tasha. Around the Year. Henry Z. Walck, 1957.

Effective pictures and verse tell about each month of the year.

__________. 1 is One. Henry Z. Walck, 1956.

A counting book from 1 to 20 with original verses.

Ungerer, Tomi. One, Two, Where's My Shoe? Harper and Row, 1964.

Basic book of form and design shown in colorful illustrations.

__________. Snail, Where Are You? Harper and Row, 1962.

Basic book of form and design will show a new world of recognition to the youngest child as the snail is found in many places.

Waller, Leslie. A Book to Begin on: Numbers. Holt, Rinehart and Winston, 1960.

Book about concept of numbers. Describes range of uses from counting change to scientific applications. Simple, direct, and useful.

__________. A Book to Begin on: Time. Holt, Rinehart and Winston, 1959.

Story of the history of time and why we divide our time into days, seasons, years.

Watson, Nancy. Annie's Spending Spree. Viking Press, 1957.

A dollar bill given to Annie for her birthday is shown to equal 2 half-dollars, 4 quarters, 10 dimes, 20 nickels, or 100 pennies. Children enjoy wondering how she will spend her 100 pennies and pantomime spending theirs.

__________. What is One? A. A. Knopf, 1954.

Charming book with illustrations to acquaint children with numbers and counting.

Webber, Irma. It Looks Like This. Wm. R. Scott, 1958.

Possibilities in this book are present for development of the kind of objectivity needed in scientific thinking.

Wondriska, William. 1, 2, 3, A Book to See. Pantheon Press, 1959.

Number symbols from 1 through 10 are illustrated by large, clear figures. Offers material for development of group recognition and counting.

Yolen, Jane H. See This Little Line? David McKay, 1963.

An introduction to drawing lines for the very young. Simple rhymed text. Fun for all.

YOUNG OWL Reading Time Series. Holt, Rinehart and Winston, 1964.

Carpenter, Walter S. Two Knots on a Counting Rope.

Development of counting, 1 through 10, is presented by correlating horses and knots on the Navaho counting rope. Indian number words are given.

Fehr, Howard F. If You Can Count to 10.

Concepts of addition, subtraction and multiplication are developed with numbers 1 to 10. Presentation of ordinal numbers, in word and figure, associated with familiar objects for young children.

Ford, Henry W. Fun With the Calendar.

Develops children's concept of measurement of time. Shows how to read claendar, learn days of the week and months of the year.

Grant, Eldon. Twenty White Horses, A Book of Division.

Concept of division developed by using groups of horses in both word and numeral form. Shows geometric backgrounds of shapes.

Jacobs, Leland B. Delight In Number.

Number concepts developed through illustrations of groups of objects with accompanying verse. Good use of number words is presented.

Nohelty, Sally. Eleven and Three Are Poetry.

Familiar rhymes present concepts of addition and subtraction in form of cardinal numbers from 1 to 10.

Rossetti, Christina. Adding: A Poem.

Groups are used to develop concept of addition facts (___and___are 2; ___and___are 4; etc.). Rhyming text accompanies this development.

Simon, Leonard. Counting Lightly.

Various ways to count objects are shown: one-to-one relationship by using rocks, sticks, notches on a stick, and finally by using fingers on the hand, with words to "count lightly."

__________. Stretching Numbers.

Concept to develop distance measurement is shown through use of different measuring instruments.

Wright, H. R. Four Threes Are 12.

Development of multiplication skills comes by showing groups of objects.

__________. A Maker of Boxes.

Concepts of shapes and sizes are developed. Use of descriptive words: round, square, long, short.

Zarchy, Harry. Wheel of Time. Crowell, 1957

An interesting development of many concepts related to time and its measurement.

Zim, Herbert. The Universe. Wm. R. Morrow, 1961.

A combination of measurement and geometry showing the earth in relation to other planets.

Ziner, Feenie. Counting Carnival. Coward-McCann, 1962.

Simple counting rhymes and illustrations show the fun of developing number comcepts 1 to 12.

__________. The True Book of Time. Children's Press, 1956.

Excellent book about time and its measurement: sun's position in the sky, shadow of a stick, sundial, burning candle, sand in an hour glass, two hands of a clock.

Zolotow, Charlotte. One Step Two..... Lothrop, Lee, and Shepard, 1955.

Little Ellen takes a walk, with her steps being counted from 1 through 12. Illustrated.

__________. Over and Over. Harper, 1957.

Shows the length of the year through explanation of exciting events in the seasons.

2. Grades 4 - 6

Adler, Irving. The Magic House of Numbers. New American Library, 1957.

Discusses in entertaining fashion the natural numbers and operations on them. Mathematical puzzles, card tricks, and number games are offered to intrigue the reader.

__________. Numbers Old and New. John Day, 1960.

Presents primitive methods of counting. Superstitions regarding numbers are presented in one chapter while another covers information on numbers in nature.

__________. Numerals: New Dresses for Old Numbers. John Day, 1964.

Newest ways of counting. Excellent treatment of place value and systems in bases other than ten.

Andrews, F. Emerson. Numbers, Please. Little, Brown and Company, 1961.

Book about number language which uses ten digits. Included are short-cuts and tricks to help understand and have fun with numbers.

Archer, A. Number Principles and Patterns. Ginn and Company, 1961.

Discussion of non-negative number systems through set of rationals and basic properties of these numbers. Probably most useful for pre-first year algebra students.

Arnold, Pauline. Money. Holiday House, 1962.

Discussion of budgeting, savings, and other forms of investments, banks, insurance, wise borrowing or spending, and inflation or deflation of money. Excellent glossary.

Asimov, Isaac. Realm of Algebra. Houghton Mifflin, 1961.

Concepts of algebra are presented in understandable terms to students without particular aptitude. Simple beginnings go to fairly advanced conclusions.

__________. Realm of Measure. Houghton Mifflin, 1960.

An explanation of the underlying theories of measurement.

Bell, Thelma. The Riddle of Time. Viking Press, 1963.

Measurement of time from Babylonian lunar months to present-day atomic clocks. Book also deals with biological aspect of time -- heartbeats, life cycles of amoeba and man -- and geological time.

Bendick, Jeanne. All Around You. McGraw-Hill Book Company, 1951.

Science picture book describes the why and how of the world around us with simple pictures and vocabulary of comparison, number, and measurement.

__________. The First Book of Time. Franklin Watts, 1963.

Story of time--seasons, hours, minutes, seconds, calendars, clocks--from dials to Einstein. Up-to-date treatment of the modern concept of time.

__________. How Much and How Many, the Story of Weights and Measures. Whittlesey House, 1960.

Up-dated revision of an earlier book to include measurements used in rocketry, atomic science, and space exploration. Includes original facts on weights and measures and how they got that way.

__________. Take a Number. Whittlesey House, 1961.

Facts, ideas, and puzzles are presented to explain numbers from finger counting to computers.

__________. Take Shapes, Lines and Letters. Whittlesey House, 1962.

Useful as a supplement to mathematics in upper elementary grades.

Buehr, Walter. Keeping Time. G.P. Putnam's Sons, 1960.

Sundials, water clocks, sand clocks, candle clocks, and the mechanical clock have been developed to help man tell time by minutes, hours, days, weeks, months, and years. Importance of the stars in development of nautical chronometer is told.

Carona, Philip. Things That Measure. Prentice-Hall, Inc., 1962.

History of measuring devices briefly sketched from origin of ruler to instruments that can measure speck of dust. Included are some eary-to-read and easy-to-understand experiments.

Gardner, Martin. Mathematical Puzzles. Crowell, 1961.

Arithmetic, money, plane and solid geometry puzzles and how to work them are given. Work goes from simple to complex to introduce a variety of principles.

Hengensbaugh, Jane. I Live In So Many Places. Grosset and Dunlap, 1956.

Relative position is developed by a child who tells of living in a house on a street in a county of a state in the United States on the North American continent in the Western Hemisphere on the earth, part of the universe.

Hine, Al. Money Round the World. Harcourt, Brace and World, 1963.

Traces development of money from barter system to use of metal coins and paper money.

Hitte, Kathryn. Hurricanes, Tornadoes, and Blizzards. Random House, 1960.

Prediction of direction of major storms is a mathematical problem. Good charting of dimensional information with easy reading text and diagrams.

Jonas, Arthur. New Ways in Math. Prentice-Hall, Inc., 1962.

Here is the story of mathematics in set theory, binary notation, probability, and algebra shown by cartoons.

Kenyon, Raymond G. I Can Learn About Calculators and Computers. Harper and Row, 1961.

Interesting account of the development of written number systems and how early man learned to count and record his calculations. Simple computing devices, including slide rule, can be made from instructions given.

Lauber, Patricia. The Story of Numbers. Random House, 1961.

Story of our number system and its uses. Chart showing names for very large numbers is interesting.

YOUNG OWL Reading Time Series. Holt, Rinehart and Winston, 1964.

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Zim, Herbert. The Universe. Wm. R. Morrow, 1961.

A combination of measurement and geometry showing the earth in relation to other planets.

Suggested Professional Books

The following are recommended for the professional libraries of elementary schools:

1. Adler, Irving. A New Look at Arithmetic. John Day, 1964.

   Aid to the elementary teacher relearning arithmetic from the modern point of view.

2. Allendoerfer, Carl B. and Oakley, C.O. Principles of Mathematics. 2nd ed. McGraw-Hill, 1963.

   Written to up-date undergraduate courses in mathematics to take due advantage of the advances made during the last century. Written as a college text. Elementary teachers may find this an invaluable self-teaching tool to deepen and up-date their knowledge of mathematics.

3. Banks, J. Houston. Learning and Teaching Arithmetic. Allyn and Bacon, 1959.

   Useful for in-service growth. Chapters 2,4,6,8, and 10 are written to develop greater insight into elementary arithmetic. Following each of these chapters there is a chapter on teaching the same aspect of arithmetic to elementary school children.

4. Bell, Clifford, and Others. Fundementals of Arithmetic For Teachers, John Wiley, 1962.

   A complete arithmetic text for teachers oriented toward a conventional elementary school curriculum with introduction to algebra and geometry. Transitional.

5. Bley, Edgar S. Math Without Numbers. Sterling, 1961.

   Designed to intrigue the "number shy", this book offers simple introduction to geometry and set theory.

6. Breuer, Joseph. Introduction to the Theory of Sets. Prentice-Hall, 1958.

   Uses observation of the concrete to introduce the subject, gradually developing properties and principles concerning sets. Leads from concrete finite sets, to cardinal numbers, to infinite cardinals, and then to ordinals.

7. Buckingham, Burdette R. Elementary Arithmetic, Its Meaning and Practice. Ginn, 1953.

   A rich source of material on arithmetic computation, providing backgrounds and understandings. Basic structure of arithmetic and modern terminology are not included.

8. Davis, Robert B. *Discovery in Mathematics*. Addison-Wesley, 1964.

Based on the Madison Project, the student booklets contain worthwhile supplementary enrichment experiences emphasizing self-discovery. Text for teachers contains extensive teachers aids, ideas for exploration, and basic concept development.

9. Grossnickle, Foster E. and Brueckner, L. J. *Discovering Meanings in Elementary School Mathematics*, 4th ed. Holt, Rinehart and Winston, 1963.

10. Marks, John L. *Teaching Arithmetic for Understanding*. McGraw-Hill, 1958.

For elementary school teachers who seek guidance in helping students develop understanding of arithmetic. Stresses material to aid the teacher to thoroughly comprehend mathematical ideas and mathematical aspects of elementary school arithmetic. Useful general methods book for the transition period.

11. National Council of Teachers of Mathematics. *Enrichment Mathematics for the Grades*, Twenty-Seventh Yearbook, The Council, 1963.

A valuable resource for teachers who need to provide for the talented. Offers enrichment units to be taken up by a class or group, articles that can be read and studied by individual students, and extracurricular activities for students.

12. National Council of Teachers of Mathematics. *Evaluation in Mathematics*, Twenty-Sixth Yearbook, The Council, 1961.

Applies basic principles of measurement to the evaluation of mathematics achievement from Grade 1 through Grade 14. Furnishes specific examples for building tests and offers help in interpreting scores and evaluating curriculum.

13. National Council of Teachers of Mathematics. *The Growth of Mathematical Ideas, Grades K-12*, Twenty-Fourth Yearbook, The Council. 1959.

Highlights the most basic mathematical themes running spirally grades from K-12. Offers key concepts of modern teaching techniques, illustrating classroom procedures. The 23rd yearbook, *Insights Into Modern Mathematics*, may also prove useful to the teacher.

14. Spitzer, Herbert F. *The Teaching of Arithmetic*, 3rd. ed. Houghton-Mifflin, 1961.

Major attention is given to classroom learning rather than the nature of arithmetic. Stresses a developmental procedure to student's solution.

15. Spitzer, Herbert F. *Practical Classroom Procedures for Enriching Arithmetic*. Webster Publishing Co., 1956.

A handy useful reference for suggestive enrichment activities.

16. Van Engen, Henry and Hartung, Maurice. The Theory of Arithmetic. Scott Foresman, 1960.

Ranges from a development of the real numbers to informal geometry. Includes discussions on the psychology, philosophy, and methods of teaching elementary school mathematics. Relates theory to algorithms.

17. Ward, Morgan and Hardgrove, Clarence E. Modern Elementary Mathematics. Addison-Wesley, 1964.

Written primarily for pre-service and in-service elementary teachers who have some knowledge of this field. Patterns underlying arithmetic and geometry are presented through inductive techniques of discovery. Uses the language and ideas of set theory throughout.

Suggested Professional Magazines

1. Arithmetic Teacher. National Council of Teachers of Mathematics.
2. Mathematics Teacher. National Council of Teachers of Mathematics.
3. Mathematics Student Journal. National Council of Teachers of Mathematics.
4. School Mathematics Study Group Newsletter. Stanford University. School of Education.
5. School Science and Mathematics. Central Association of Science and Mathematics Teachers, Inc.
6. Science and Math Weekly. American Education Publication.

Precision in Vocabulary

One of the strongest keystones of mathematical efficiency lies in its vocabulary. The mathematician says things precisely, accurately, and briefly. Probably one of the greatest quarrels the mathematics teacher may have with the elementary level teaching today is vocabulary. The mathematics teacher can have no use for vocabulary that is not precise and accurate. Precision in vocabulary is necessary for understanding and meaning of the concept or process being learned.

The words listed below are the essential vocabulary for the indicated grade levels. After each word has been introduced, its meaning is to be maintained and extended at each succeeding grade level.

The vocabulary list, for your convenience, has been divided into two parts. Part I contains all of the words introduced in the primary grades (1-3). Part II contains all of the words introduced in the intermediate grades (4-6).

Part I is organized around the following content areas:

A. Numeration
B. Whole Numbers
C. Geometry
D. Measurement and Money

Part II is organized around the following content areas:

A. Numeration
B. Whole Numbers
C. Rational Numbers
D. Geometry
E. Measurement and Money
F. Tables, Graphs, and Scale Drawings

Part I

NUMERATION

	Readiness	1st	2nd	3rd	4th	5th	6th
above	x---	---	---	---	---	---	---
after		x---	---	---	---	---	---
algorithm		x---	---	---	---	---	---
alike	x---	---	---	---	---	---	---
answer				x---	---	---	---
arabic numeral			x---	---	---	---	---
before		x---	---	---	---	---	---
below	x---	---	---	---	---	---	---
beside	x---	---	---	---	---	---	---
between		x---	---	---	---	---	---
bigger	x---	---	---	---	---	---	---
biggest	x---	---	---	---	---	---	---
binary operation				x---	---	---	---
bottom	x---	---	---	---	---	---	---
borrowing			x---	---	---	---	---
braces				x---	---	---	---
carrying			x---	---	---	---	---
collection		x---	---	---	---	---	---
combination		x---	---	---	---	---	---
count	x---	---	---	---	---	---	---
different	x---	---	---	---	---	---	---
difference		x---	---	---	---	---	---
digit		x---	---	---	---	---	---
doubles			x---	---	---	---	---
eight	x---	---	---	---	---	---	---
eighty		x---	---	---	---	---	---
empty set	x---	---	---	---	---	---	---
equals (=)		x---	---	---	---	---	---
equation		x---	---	---	---	---	---
equivalent set	x---	---	---	---	---	---	---
even number			x---	---	---	---	---
expanded notation		x---	---	---	---	---	---
fewer	x---	---	---	---	---	---	---
fewest	x---	---	---	---	---	---	---
fifty		x---	---	---	---	---	---
finite set							
first	x---	---	---	---	---	---	---
five	x---	---	---	---	---	---	---
forty		x---	---	---	---	---	---
four	x---	---	---	---	---	---	---
fourth		x---	---	---	---	---	---
greater	x---	---	---	---	---	---	---
greater than ( )		x---	---	---	---	---	---
greatest	x---	---	---	---	---	---	---
grouping principle		x---	---	---	---	---	---
half		x---	---	---	---	---	---
height				x---	---	---	---
higher		x---	---	---	---	---	---

	Readness	1st	2nd	3rd	4th	5th	6th
highest		x----	----	----	----	----	----
hundreds			x----	----	----	----	----
identity element							
a. addition		x----	----	----	----	----	----
b. multiplication			x----	----	----	----	----
inequality		x----	----	----	----	----	----
infinite set		x----	----	----	----	----	----
intersection (ing)		x----	----	----	----	----	----
in	x----	----	----	----	----	----	----
last	x----	----	----	----	----	----	----
larger	x----	----	----	----	----	----	----
largest	x----	----	----	----	----	----	----
least	x----	----	----	----	----	----	----
left (column)				x----	----	----	----
less	x----	----	----	----	----	----	----
lesser	x----	----	----	----	----	----	----
less than ( )		x----	----	----	----	----	----
lower	x----	----	----	----	----	----	----
lowest	x----	----	----	----	----	----	----
minus		x----	----	----	----	----	----
natural number		x----	----	----	----	----	----
near doubles				x----	----	----	----
nine	x----	----	----	----	----	----	----
ninety		x----	----	----	----	----	----
not any	x----	----	----	----	----	----	----
number base							
number		x----	----	----	----	----	----
number pattern		x----	----	----	----	----	----
number line		x----	----	----	----	----	----
number properties			x----	----	----	----	----
numeral		x----	----	----	----	----	----
numeration			x----	----	----	----	----
one	x----	----	----	----	----	----	----
one more	x----	----	----	----	----	----	----
one-to-one matching	x----	----	----	----	----	----	----
odd number			x----	----	----	----	----
open sentence			x----	----	----	----	----
order principle		x----	----	----	----	----	----
over	x----	----	----	----	----	----	----
pair		x----	----	----	----	----	----
pattern	x----	----	----	----	----	----	----
parentheses ( )		x----	----	----	----	----	----
place value		x----	----	----	----	----	----
plus (+)			x----	----	----	----	----
rational number		x----	----	----	----	----	----
reverse				x----	----	----	----
roman numeral		x----	----	----	----	----	----
row	x----	----	----	----	----	----	----
same number as		x----	----	----	----	----	----
second	x----	----	----	----	----	----	----
set	x----	----	----	----	----	----	----
seven	x----	----	----	----	----	----	----

	Readiness	1st	2nd	3rd	4th	5th	6th
seventy		x----	-----	-----	-----	-----	-----
skip counting		x----	-----	-----	-----	-----	-----
six	x----	-----	-----	-----	-----	-----	-----
sixty		x----	-----	-----	-----	-----	-----
solve		x----	-----	-----	-----	-----	-----
subset		x----	-----	-----	-----	-----	-----
ten		x----	-----	-----	-----	-----	-----
third	x----	-----	-----	-----	-----	-----	-----
thirty		x----	-----	-----	-----	-----	-----
three	x----	-----	-----	-----	-----	-----	-----
twenty		x----	-----	-----	-----	-----	-----
two	x----	-----	-----	-----	-----	-----	-----
thousand (s)			x----	-----	-----	-----	-----
times			x----	-----	-----	-----	-----
under	x----	-----	-----	-----	-----	-----	-----
union of sets		x----	-----	-----	-----	-----	-----
variable				x----	-----	-----	-----
zero	x----	-----	-----	-----	-----	-----	-----

	Readiness	1st	2nd	3rd	4th	5th	6th
WHOLE NUMBERS							
add		x	-	-	-	-	-
addend			x	-	-	-	-
addition		x	-	-	-	-	-
addition table			x	-	-	-	-
borrowing			x	-	-	-	-
carrying			x	-	-	-	-
counting numbers	x	-	-	-	-	-	-
difference		x	-	-	-	-	-
digit		x	-	-	-	-	-
divide		x	-	-	-	-	-
multiplication			x	-	-	-	-
multiply (x)				x	-	-	-
number facts				x	-	-	-
number ladder				x	-	-	-
number line		x	-	-	-	-	-
order principle			x	-	-	-	-
place value			x	-	-	-	-
product			x	-	-	-	-
remainder				x	-	-	-
subtract		x	-	-	-	-	-
sum		x	-	-	-	-	-
subtraction			x	-	-	-	-
times			x	-	-	-	-

	Readiness	1st	2nd	3rd	4th	5th	6th
GEOMETRY							
angle				x---	------	------	------
circle	x---	------	------	------	------	------	------
closed space				x---	------	------	------
geometry		x---	------	------	------	------	------
height				x---	------	------	------
inside (figure)		x---	------	------	------	------	------
outside (figure)		x---	------	------	------	------	------
plane				x---	------	------	------
ray				x---	------	------	------
rectangle		x---	------	------	------	------	------
segment				x---	------	------	------
square		x---	------	------	------	------	------
triangle		x---	------	------	------	------	------

	Readiness	1st	2nd	3rd	4th	5th	6th
MEASUREMENT & MONEY							
bottom	x-----	-----	-----	-----	-----	-----	-----
cent		x-----	-----	-----	-----	-----	-----
centimeter		x-----	-----	-----	-----	-----	-----
change (money)		x-----	-----	-----	-----	-----	-----
cup		x-----	-----	-----	-----	-----	-----
degree (°)		x-----	-----	-----	-----	-----	-----
dime		x-----	-----	-----	-----	-----	-----
dollar			x-----	-----	-----	-----	-----
foot		x-----	-----	-----	-----	-----	-----
gallon			x-----	-----	-----	-----	-----
half			x-----	-----	-----	-----	-----
half dollar			x-----	-----	-----	-----	-----
inch		x-----	-----	-----	-----	-----	-----
length			x-----	-----	-----	-----	-----
less		x-----	-----	-----	-----	-----	-----
linear measure		x-----	-----	-----	-----	-----	-----
liquid measure		x-----	-----	-----	-----	-----	-----
longer	x-----	-----	-----	-----	-----	-----	-----
longest	x-----	-----	-----	-----	-----	-----	-----
many	x-----	-----	-----	-----	-----	-----	-----
middle		x-----	-----	-----	-----	-----	-----
money		x-----	-----	-----	-----	-----	-----
more	x-----	-----	-----	-----	-----	-----	-----
most	x-----	-----	-----	-----	-----	-----	-----
nickel		x-----	-----	-----	-----	-----	-----
penny		x-----	-----	-----	-----	-----	-----
pint		x-----	-----	-----	-----	-----	-----
pound			x-----	-----	-----	-----	-----
quart			x-----	-----	-----	-----	-----
quarter			x-----	-----	-----	-----	-----
scale			x-----	-----	-----	-----	-----
shorter	x-----	-----	-----	-----	-----	-----	-----
shortest	x-----	-----	-----	-----	-----	-----	-----
smaller	x-----	-----	-----	-----	-----	-----	-----
smallest	x-----	-----	-----	-----	-----	-----	-----
taller	x-----	-----	-----	-----	-----	-----	-----
tallest	x-----	-----	-----	-----	-----	-----	-----
temperature		x-----	-----	-----	-----	-----	-----
thermometer		x-----	-----	-----	-----	-----	-----
top	x-----	-----	-----	-----	-----	-----	-----
twice			x-----	-----	-----	-----	-----
under	x-----	-----	-----	-----	-----	-----	-----
width				x-----	-----	-----	-----
yard		x-----	-----	-----	-----	-----	-----

PART II

	4th	5th	6th
NUMERATION			
abacus	x------------------		
base 5		x----------	
base 8			x--
billion		x----------	
cardinal	x------------------		
closure prop.			
a. addition			x--
b. multiplication			x--
decimal	x------------------		
decimal point	x------------------		
decimal numeral	x------------------		
diagonal	x------------------		
disjoint set			x--
divisibility	x------------------		
equality sign	x------------------		
equivalent equations	x------------------		
equivalent fractions	x------------------		
equivalent numerals	x------------------		
exponent			x--
Hindu-arabic system	x------------------		
holds a place	x------------------		
inequalities	x------------------		
is equal to	x------------------		
is greater than	x------------------		
is less than	x------------------		
is not equal to	x------------------		
is not less than		x----------	
is not greater than	x------------------		
million	x------------------		
minuend	x------------------		
number sentence	x------------------		
open sentence	x------------------		
ordinal	x------------------		
percent		x----------	
period	x------------------		
powers of ten			x--
regroup	x------------------		
rename	x------------------		
replacement set			x--
simplest name	x------------------		
subtrahend	x------------------		
solution set			x--
undoes	x------------------		
whole numbers		x----------	

	4th	5th	6th
WHOLE NUMBERS			
array	x-------	----------	--------
associative operation		x-------	--------
assoc. prop. addition		x-------	--------
assoc. prop. multip.		x-------	--------
average	x-------	----------	--------
closed			x--
commutative		x-------	--------
commutative operation		x-------	--------
comm. prop. addition	x-------	----------	--------
comm. prop. multip.		x-------	--------
composite number			x--
distributive prop.		x-------	--------
dividend		x-------	--------
division	x-------	----------	--------
doing (add)		x-------	--------
elements	x-------	----------	--------
estimate	x-------	----------	--------
factor	x-------	----------	--------
grouping	x-------	----------	--------
integer			x--
inverse operation			x--
indent		x-------	--------
is approx = to			x--
lattice method			x--
micrometer caliper			x--
minuend	x-------	----------	--------
negative interger			x--
negative number			x--
partial product	x-------	----------	--------
place holder	x-------	----------	--------
positive integer			x--
positive number			x--
prime factor			x--
prime integer			x--
product	x-------	----------	--------
quotient	x-------	----------	--------
remainder	x-------	----------	--------
short division		x-------	--------
set of whole numbers	x-------	----------	--------
test of divisibility			x--
unlimited number		x-------	--------

RATIONAL NUMBERS	4th	5th	6th
caret		x------	------
commission		x------	------
common denominator		x------	------
common divisor		x------	------
common multiple		x------	------
complex fraction		x------	------
denominator	x------	------	------
decimal numeral	x------	------	------
decimal point	x------	------	------
discount		x------	------
divisible		x------	------
divisor		x------	------
eighths	x------	------	------
equivalent decimals	x------	------	------
equivalent fractions	x------	------	------
fraction	x------	------	------
fraction bar	x------	------	------
fractional parts	x------	------	------
interest			x--
improper fraction		x------	------
least common denominator		x------	------
least common multiple		x------	------
lowest terms	x------	------	------
mixed numeral	x------	------	------
mark-up		x------	------
numerator	x------	------	------
percent	x------	------	------
principal			x--
proportion			x--
rate		x------	------
ratio			x--
terms	x------	------	------
time			x--
proper fraction		x------	------
reciprocals		x------	------
simplify (simplest terms)		x------	------

	4th	5th	6th
GEOMETRY			
alternate interior angle			x--
angle	x-------	--------	--------
base	x-------	--------	--------
bisecting an angle		x-------	--------
cylinder	x-------	--------	--------
cube	x-------	--------	--------
cone			x--
closed plane figure	x-------	--------	--------
curves	x-------	--------	--------
congruent		x-------	--------
copying a line segment		x-------	--------
construct		x-------	--------
center	x-------	--------	--------
circumference		x-------	--------
central angle		x-------	--------
copying an angle			x--
corresponding angles			x--
common side		x-------	--------
diameter	x-------	--------	--------
diameter length		x-------	--------
distance	x-------	--------	--------
decagon			x--
edges	x-------	--------	--------
endpoint (s)	x-------	--------	--------
equilateral		x-------	--------
exterior	x-------	--------	--------
faces		x-------	--------
English system of measure	x-------	--------	--------
four-sided figure	x-------	--------	--------
flat surface	x-------	--------	--------
formula		x-------	--------
geometry	x-------	--------	--------
heptagon			x--
hexagon			x--
intersecting		x-------	--------
isoceles			x--
inside	x-------	--------	--------
line	x-------	--------	--------
line segment	x-------	--------	--------
location	x-------	--------	--------
metric system			x--
octagon			x--
outside	x-------	--------	--------
on	x-------	--------	--------
parallel lines	x-------	--------	--------
plane figure	x-------	--------	--------
Pi =		x-------	--------
points	x-------	--------	--------
pyramid			x--
perimeter	x-------	--------	--------
parallelogram	x-------	--------	--------
perpendicular lines		x-------	--------
quadrilateral			x--
ray	x-------	--------	--------
rectangle	x-------	--------	--------

	4th	5th	6th
radius	x------	------	------
radii	x------	------	------
rhombus	x------	------	------
right angle		x------	------
right triangle			x--
right prism			x--
right cylinder			x--
space	x------	------	------
sphere	x------	------	------
side		x------	------
supplementary angle		x------	------
sector		x------	------
standard lengths of length	x------	------	------
space figure			x--
triangle	x------	------	------
trapezoid			x--
vertical (angle)	x------	------	------
vertex	x------	------	------
pentagon	x------	------	------

	4th	5th	6th
MEASUREMENT AND MONEY			
area	x------	-------	-------
base			x--
cubic inch			x--
dimensions	x------	-------	-------
foot	x------	-------	-------
feet		x------	-------
height		x------	-------
inch	x------	-------	-------
length	x------	-------	-------
linear measurement	x------	-------	-------
mile	x------	-------	-------
square inch		x------	-------
square foot		x------	-------
square mile	x------	-------	-------
square yard	x------	-------	-------
surface area	x------	-------	-------
volume			x--
width	x------	-------	-------

TABLES, GRAPHS AND SCALE DRAWINGS

	4th	5th	6th
bar graph		x------------	
centimeter			x--
circle graph		x------------	
data			x--
decimeter			x--
horizontal bar graph			x--
kilometer			x--
line graph		x------------	
line segment graph		x------------	
meter			x--
metric **system**			x--
scale		x------------	
scale drawings		x------------	
standards units of measure			x--
table		x------------	
vertical bar graph			x--